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Research article

Incompressible Fluid flow through free and porous areas

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Abstract:
In this paper we discussedincompressiblefluid flow problem through free and porous areas by using Darcy's law and continuity equation, by apply the boundary conditions required to specify the solution

Keywords: Incompressible fluid**,** Darcy's law**,** continuity equation, pressure, velocity

1-Introduction:

Consider a porous sphere of a radius a with constant permeability k , its centreconcedes with the coordinate origin , concentric with another spherical shell with inner radius b (b > a) (SaifAlDien 2017) .consider a viscous incompressible fluid with low Reynolds number moves uniformly with constant velocity U past the body , and that the porous regions saturated with the fluid (Clattery 1967). Boundary conditions provide information about the behavior of the solution at physical boundaries of the domain(Quintard and Whitaker 1994) . In general three types of the boundary conditions are distinguished(Hilfer 1998) 1. Dirichlet boundary conditions specify the values of the solution at a given part of the boundary; 2. Neumann boundary conditions specify the value of the spatial derivative of the solution. In the case of flow in porous media, this type of condition is usually written in terms of the fluid flux in the direction normal to the boundary; 3-(Mokadam1961). Robin boundary conditions, which specify a relationship between the value of solution and its derivative, and can be viewed as a generalization of both Dirichlet and Neumann boundary conditions(Dagan 1989). The boundary conditions required to specify the solution are : 1-The continuity of pressures of different regions at the interface boundary. 2- The continuity of normal velocities.

3- $-\beta(u_\theta - q_\theta) = \frac{\partial u_r}{\partial r} + \frac{1}{R_\theta}\left[\frac{\partial u_r}{\partial \theta} - u_\theta\right] \; at\; r = b \quad and$

$\beta(u_\theta - q_\theta) = \frac{\partial u_\theta}{\partial r} + \frac{1}{R_\theta}\left[\frac{\partial u_r}{\partial \theta} - u_\theta\right] \; at\; r = a$

Where $\beta = \frac{\alpha}{\sqrt{k}}$ R_θ is radius of curvature

4-The flow of each fluid can be described by the extended Darcy formula including the relative permeability coefficient.

5-Both fluid phases are bar tropic, i.e. each phase density depends only on the pressure in the respective phase(Brenner 1980)

2- Mathematical formulation:

The porous regions (regions 1 and region 3)apply Darcy's law

$q = -\frac{k}{\gamma}\nabla p$ (1)

Together with the continuity equation

$\nabla . q = 0$ (2)

Assume a trial solution in the the form

$\varphi(r,\theta) = \mu\, U\, g(r) cos\theta$ (3)

The velocity components

$$u_r = U\left[1 + \frac{A}{r^3} + \frac{B}{r} + Dr^2\right]cos\theta$$

$$u_\theta = -U\left[1 - \frac{A}{2r^3} + \frac{B}{2r} + 2Dr^2\right]sin\theta$$
(4)

Then the pressure is given by

$$p(r,\theta) = p_\infty + \gamma\, U\left[Er + F\,\frac{1}{r^2}\right]cos\theta$$
5

Where E and F are arbitrary constants

3- Mathematical Solution:

To specify the pressure for region 1 and 3, we have to take into consideration the following conditions:

$|g(r)| \to 0 \quad as \;\; r \to \infty$

$|g(r)| < \infty \quad as \;\; r \to 0$ (6)

Now from 5 and 6 we have the pressure for regions 1 and 3 given respectively by:

Region 1 :

$p(r,\theta) = p_\infty + \gamma\, UEr\, cos\theta$

Region 3:

$p(r,\theta) = p_\infty + \gamma\left[\frac{U\,F}{r^2}\right]cos\theta$ (7)

Now from spherical polar coordinates

$$\nabla p = \frac{\partial p}{\partial r} i_{-r} + \frac{1}{r}\frac{\partial p}{\partial \theta} i_{-\theta}$$

And

q= $i_{-r}q_r + i_{-\theta}q_\theta$ (8)

Where $i_{-r} and\; i_{-\theta}$ are local unit vectors.

Comparing equations 1 and 8 we have

$$q_r = -\frac{k}{\mu}\frac{\partial p}{\partial r}$$

$q_\theta = -\frac{k}{r\,\mu}\frac{\partial p}{\partial \theta}$ (9)

By using equations (7) and (9) the velocity field for regions 1 and 3 given respectively given by :

Region 1:

$q_r = -\,k\, E\, Ucos\theta$

$q_\theta = k\ E\ U\ sin\theta$ (10)

Region 3:

$$q_r = \frac{2k\ F}{\mu}\ Ucos\theta$$

$$q_\theta = \frac{2k\ F}{r^2}\ sin\theta$$

To find the constants A,B,D,E and F we apply the boundary conditions between regions I and II at r = a , and between regions II and III at r = b

Matching boundary conditions between regions I and II (r= a) we have the following

$$u_r(r,\theta) = q_r(r,\theta) \rightarrow A + a^2 B + a^5 D + ka^3 E = -a^3$$

$$p_I(r,\theta) = p_{II}(r,\theta) \rightarrow B + 10a^3 D - a^3 E = 0$$

$$\beta(u_\theta - q_\theta) = \frac{\partial u_\theta}{\partial r} + \frac{1}{R_\theta}\left[\frac{\partial u_r}{\partial \theta} - u_\theta\right] \rightarrow$$

$$(2\beta a - 10)A - \beta a^3 B - (6a^3 + 4\beta a^6)D - 2\beta a^4 E = 2\beta a^4 \qquad (11)$$

Matching boundary conditions between regions II and III (r= b) we have the following

$$u_r(r,\theta) = q_r(r,\theta) \rightarrow A + b^2 B + a^5 D - 2kF = -b^3$$

$$p_I(r,\theta) = p_{II}(r,\theta) \rightarrow B + 10b^3 D - F = 0$$

$$-\beta(u_\theta - q_\theta) = \frac{\partial u_r}{\partial r} + \frac{1}{R_\theta}\left[\frac{\partial u_r}{\partial \theta} - u_\theta\right] \rightarrow$$

$$(2\beta b - 10)A - \beta b^3 B - (4\beta b^6 - 6b^5)D + 2kbF - 2\beta BF = 2\beta b^4 \qquad (12)$$

By Solve equations (11) and (12) to get the value of the constants A , B, D, E, F

4- Conclusion :

In this problem of viscous fluid flow past liquid shell embedded we have three regions (porous regions I, III and free region II) by Appling the boundary conditions by considering the continuity of pressures and normal velocities at the interface boundaries and in addition taking into consideration that the difference of tangential velocities of different flow regimes are proportional to the rate of strain tensors . and get pressure and velocity

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