Dynamic Response of Railway Bogie Due to Change in Vertical Primary Suspension Stiffness Parameters

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Abstract

A mathematical model of a railway carriage moving on tangent tracks is constructed by deriving the equations of motion concerning the model in which single-point and two-point wheel-rail contact is considered. The presented railway carriage model comprises of carbody and front and rear simple conventional bogie with two leading and trailing wheelets attached to each bogie. The railway carriage is modeled by 31 degrees of freedom which govern vertical displacement, lateral displacement, roll angle and yaw angle dynamic response of wheelset whereas vertical displacement, lateral displacement, roll angle, pitch angle and yaw angle dynamic response of carbody and each of the two bogies. Linear stiffness and damping parameters of longitudinal, lateral and vertical primary and secondary suspensions are provided to the railway carriage model. Combination of linear Kalker's theory and nonlinear Heuristic model is adopted to calculate the creep forces in which introduced at wheel and rail contact patch area. Computer aided-simulation is constructed to solve the governing differential equations of the mathematical model using Runge-Kutta fourth order method. Principle of limit cycle and phase plane approach is applied to realize the stability and to evaluate the concerning critical hunting velocity at which railway carriage starts to hunt. Numerical simulation model is used to study the dynamic responses of a railway carriage bogie subjected to specific parameters of wheel conicity and primary suspension characteristics. A comparison to study the sensitivity of railway carriage bogie to dynamic responses is also presented at different vertical primary suspension stiffness parameters.

Keywords: railway carriage, railway bogie, tangent tracks, wheel conicity, suspension stiffness, critical hunting velocity

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مستخلص

نموذج رياضي لعربة قطار تتحرك على خط سكة حديد مستقيمة ومستوية تم بنائها عن طريق اشتقاق المعادلات الرياضية الخاصة بحركة العربة النموذج مع الاخذ بنظر الاعتبار نظام منفرد ومزدوج لنقاط اتصال دواليب عربة القطار مع سكة الحديد. النموذج الحالي لعربة القطار يتكون من كابينة المسافرين وعربتي دواليب امامية وخلفية مزودة كل منهما بمجموعتي دواليب امامية وخلفية ايضا. ان النموذج الرياضي متكون من 31 درجة حرية حركة والتي تمثل الازاحة العمودية والازاحة الجانبية والازاحة الدورانية حول المحور الامامي والازاحة الدورانية حول المحور العمودي بالنسبة الى مجموعة الدواليب الخاصة بعربة القطار بينما تمثل الازاحة العمودية والازاحة الجانبية والازاحة الدورانية حول المحور الامامي والازاحة الدورانية حول المحور الجانبي والازاحة الدورانية حول المحور العمودي بالنسبة الى مجموعتي كابينة المسافرين وعربتي الدواليب. ان نموذج عربة القطار جهزت بواسطة وسائل تعليق ذات صلابة نابضية و مخمدات لزوجة خطية السلوك. استخدمت نظرية ازدواجية متكونة من نظرية Kalker الخطية و نظرية Heuristic اللاخطية لغرض حساب قوى التشوية المتولدة في منطقة تلاقي دواليب العربة مع خط سكة الحديد. المعادلات الرياضية المشتقة والخاصة بنموذج عربة القطار تم ادخالها ضمن برنامج حاسوب لغرض حلها وتحديد السلوك الحركي للنموذج وباستخدام نظريات حل عددية والمسماة بنظرية Runge-Kutta العددية الرباعية . ان النموذج الحالي استخدم لغرض دراسة تاثير تغير القيم الخاصة بصلابة نابض وسائل التعليق العمودية الاولية على السلوك الحركي لنموذج عربة الدواليب وتاثير هذا التغير على السرعة الرجة لعربة القطار كما اجريت مقارنة حول مدى حساسية وتاثر الازاحات والسلوك الحركي لعربة الدواليب وتاثرها بالسرعة الحرجة التي عندها تكون عربة الدواليب في حالة الاستقرارية الحرجة.

1. Introduction

Dynamic response analysis is used to study the dynamic behavior of railway carriage bogie due to some external inputs such as rail irregularities, sudden disturbances, rail maneuvers, breaking or accelerating and other imperfections. Problems arise due to these undesirable inputs when railway carriage begin to move in different directions as vertical, pitch, roll, lateral and yaw directions. These movements cause vibrations and damage in railway components with uncomfortable ride passenger. As railway carriage laterally moves, bogies have a tendency to move in vertical direction to introduce hunt or bounce motion seriously. Pitch motion or the rotation of railway carriage about the lateral direction causes damage to the components of railway carriage due to strong coupling impacts. Pitch together with bounce motion introduce uncomfortable ride passenger in railway carriage. Roll motions or rotation about the longitudinal direction arise as the wheelsets move in the lateral direction over the track due to any external disturbances. Roll motion in railway carriage is so complicated since the critical roll motion may cause railway carriage to overturn. Lateral displacements occur due to imperfections and irregularities in the track which cause different undesirable motions like roll, yaw, and pitch. Lateral forces arise in the wheel-rail contact patch plane due to interactions between the wheel and the rail which force wheelsets to move laterally and may climb the rail. These introduced forces called creep forces in which depend upon different creep coefficients. The magnitudes of creep coefficients depend upon the wheel-rail geometry, normal load, and material properties. Many investigations used different magnitudes of creep coefficients and a combination of linear Kalker's theory [1] and nonlinear Heuristic is used in the present study. The study of railway carriage dynamic behavior should take into account the responses of railway carriage to these displacements and movements and more degrees of freedom should be considered to verify the accuracy of the system model. Different railway carriage models with different degrees of freedom are investigated and presented by many papers concerning railway carriage dynamic response. Dynamic stability of railway vehicle wheelsets and bogies having profiled wheels was presented by Wickens [2] in which two degrees of freedom model was suggested govern lateral and yaw angle of each wheelset. Nonlinear mathematical model of dynamic simulation has been established with 7 degrees of freedom by Jawahar et al. [3] which govern lateral and yaw movements of wheelsets and lateral, yaw and roll movements for both conventional and unconventional bogies. Xu et al. [4, 5] studied the dynamic analysis of coupled train-bridge systems under fluctuating wind and stated a railway carriage model that each 4-axle vehicle in a train is modeled by 27 degrees of freedom dynamic system. A vehicle model by Wang [6] was developed to represent a 23 degrees of freedom conventional freight car, consisting of a carbody, two bolsters and two truck assemblies where the carbody was assigned five degrees of freedom govern vertical lateral, yaw, pitch, and roll while each bolster was assigned three degrees of freedom vertical, lateral and roll motion. Nath et al. [7] studied the influence of yaw stiffness on the nonlinear dynamics of railway wheelset used two degrees of freedom model which govern the lateral and yaw motions. Nonlinear differential equations modeled by 8 and 10 degrees of freedom of railway carriage moving on curved tracks are presented by Lee et al. [8, 9]. Train vehicle model considered by Kumaran et al. [10] conforming to Indian railways consists of a vehicle body, two bogies with four wheelsets in which the system is modeled by 17 degrees of freedom. Mohan [11] suggested railway carriage model comprising carbody, two bogies with four wheelsets, in which the railway model assigned two degrees of freedom which govern lateral and yaw motions of each wheelset bogies whereas the full vehicle assigned three degrees of freedom which govern lateral, yaw and roll motions. Study the effects of railway track imperfections on track dynamic behavior, and the effect of unsupported sleepers on the normal load of wheel-rail were investigated by Zhang et al. [12] in which the system is modeled by 35 degrees of freedom that consider the lateral and vertical displacement, roll, pitch and yaw angle for the carbody, front and rear bogie frames and the four wheelsets. An ideal truck model with full frame decoupling represented by Dukkipati et al. [13, 14] which is modeled by 8 degrees of freedom. An investigation of dynamic interaction of long suspension bridges with running trains is presented by Xia et al. [15] in which a 27 degrees of freedom model is used. A new finite element model for three-dimensional analysis of high-speed train–bridge interactions is proposed by Song et al. [16] in which the equations of motion of the vehicle-bridge were derived using Lagrange's equation where the carbody is considered with four degrees of freedom which govern bouncing, swaying, pitching, and yawing whereas bouncing, sliding, swaying, pitching, rolling and yawing motions are considered for the bogie. Li et al. [17] investigated the problem of railway vehicle suspension estimation in which lateral and yaw modes are important and wheelsets and bogie have two degrees of freedom which govern lateral and yaw motions. A nonlinear model of a single wheelset moving with constant speed on a purely straight track is presented by Pater [18], thus the equations of motion were written down either as six equations containing the normal forces, or as four equations which do not contain the normal forces. Yugat et al. [19] presented an analytical model of wheel-rail contact force due to the passage of a railway vehicle on a curved track used equations of motion govern vertical and roll motion of right and left wheel while vertical motion of right and left rail. A nonlinear wagon-track model with 23 degrees of freedom is presented by Sun et al. [20] used to study rail corrugation formation due to the wheel stick-slip process. Rajib et al. [21] presented equations of motion govern vertical motion of front and rear wheelset, bounce and pitch motion of bogie and bounce motion of carbody to study the dynamic analysis of railway vehicle-track interactions. Railway vehicle dynamics during motion along a curved track is examined by Zboinski [22, 23] in which the dynamic behavior of the system is studied using two different methods, the quasi-statical and dynamical approach. In additional the research concerned the influence of vehicle suspension parameters as well as conditions of motion (speed, super-elevation, curve radius, transition curve existence) on limit cycle occurrence. The present study considers a railway carriage consists of carbody, two trucks and four conventional wheelsets modeled by 31-degrees of freedom which govern bounce, pitch, roll, lateral, and yaw motions of the system. The procedure done in this study is to derive the second order governing differential equations of motion of the full railway carriage and transformed these equations into a set of first order differential equations using especial technique to facilitate solving them with numerical methods. Computer-aided simulation is used to solve these equations with Runge Kutta fourth-order method to represent the dynamic behavior of the system by increasing the forward speeds of the system to reach the critical hunting velocity. Principle of limit cycle approach [11] is used to specify the critical hunting velocity of the system in which subjected to different amounts of primary suspension parameters and wheel conicity. The dynamic responses of railway carriage truck subjected to different parameters of vertical primary suspension characteristics and wheel conicities at critical hunting velocity are investigated using the constructed numerical simulation model. A comparison to study the sensitivity of railway carriage truck to dynamic responses is also presented at different vertical primary suspension stiffness parameters. The forward speed of the railway vehicle is constant at critical hunting velocity in which the railway carriage starts to hunt.

2. Mathematical Railway Carriage Model

The railway carriage is a combination of components and wheelsets joining together by a set of different primary and secondary suspension elements, in which the full railway carriage configuration model system consists of carbody, two conventional bogies, and four wheelsets as shown in **Figure 1.** A railway carriage model of 31 degrees of freedom is constructed in this research to study the dynamic responses at critical hunting velocity of railway carriage components moving on tangent tracks. The differential equations of motion govern lateral displacement Y_w, Y_b, Y_c vertical displacement Z_w, Z_b, Z_c roll angle ϕ_w, ϕ_b, ϕ_c and yaw angle ψ_w, ψ_b, ψ_c of wheelset, bogie and carbody respectively while pitch angle θ_b, θ_c of bogie and carbody. Railway carriage model is equipped with eight longitudinal, lateral and vertical primary suspensions of spring stiffness K_{px}, K_{py}, K_{pz} respectively and viscous damping constant C_{px}, C_{py}, C_{pz} respectively. Also the system is provided with eight longitudinal, lateral and vertical secondary suspensions of spring stiffness K_{sx}, K_{sy}, K_{sz} respectively and viscous damping constant C_{sx}, C_{sy}, C_{sz} respectively. Symbols and notations are illustrated in the nomenclature in **Table 1**. Dynamic behavior of railway carriage is caused by wheel-rail interactions in which creep forces are introduced at wheel-rail contact patch area. Non-conservative forces and elastic deformations at the contact patch introduce a phenomenon of creep and combination of linear Kalker's theory [1] and nonlinear Heuristic is considered to calculate the introduced creep forces. Vibrations are transmitted through connected suspensions to other railway carriage components and the dynamical behavior of the system is governed by the equations of motion of each component of railway carriage.

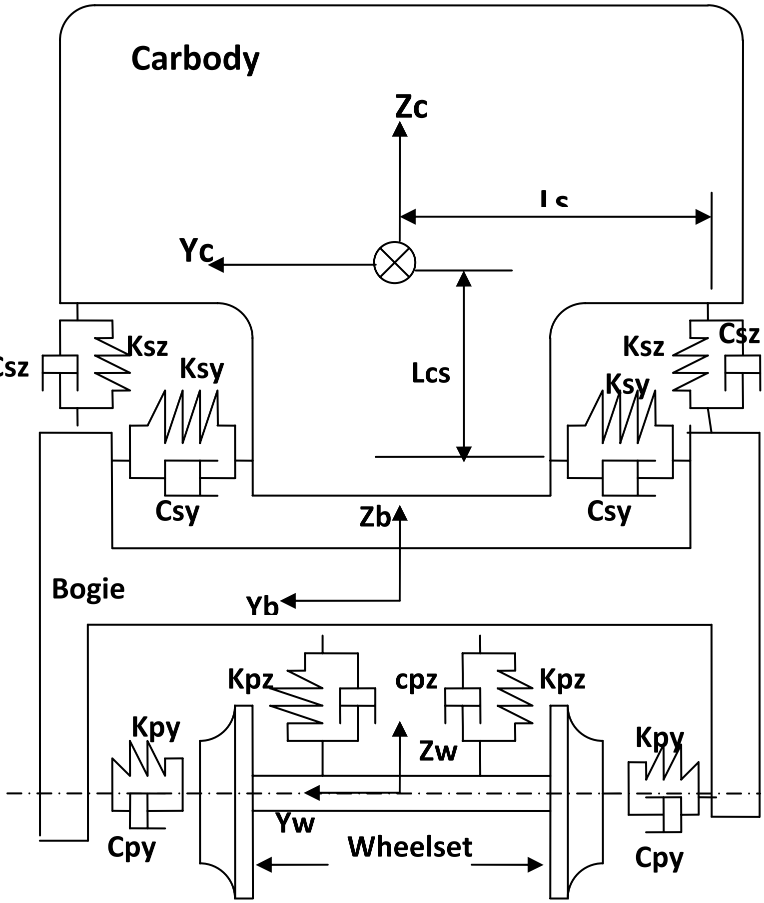

Figure (1) Front view of railway carriage components equipped with sets of primary and secondary suspension elements.

Table (1) .Nomenclature

m_w	Mass of wheelset	f_{11}	Lateral creep forces coefficient
m_b	Mass of bogie	f_{12}	Lateral-spin creep forces coefficient
m_c	Mass of carbody	f_{22}	Spin creep forces coefficient
W	Weight of wheelset	f_{33}	Longitudinal creep forces coefficient
r0	Nominal wheel rolling radius	V	Forward 6 speed of railway carriage
Y_{wi}	Lateral displacement of wheelset (i=1, 2, 3, 4)	Y_{bj}	Lateral displacement of bogie (i=1, 2)
Φ_{wi}	Roll angle displacement of wheelset (i=1, 2, 3, 4)	Φ_{bj}	Roll angle displacement of bogie (i=1, 2)
Ψ_{wi}	Yaw angle displacement of wheelset (i=1, 2, 3, 4)	Ψ_{bj}	Yaw angle displacement of bogie (i=1, 2)
Z_c	Bounce or vertical displacement of carbody	Y_c	Lateral displacement of carbody
Φ_c	Roll angle displacement of carbody	Θ_c	Pitch angle displacement of carbody
Θ_{bj}	Pitch angle displacement of bogie (i=1, 2)	Ψ_c	Yaw angle displacement of carbody
a	Half of track gauge	λ	Wheel profile conicity

2.1 Wheelsets Differential Equations of Motion

The railway carriage model is equipped with four conventional wheelsets in which consists of two wheels attached together by a solid axle as shown in **Figure 2**. Wheelsets are used to steer and support the carriage. Wheelsets equations of motion are derived using Newton's laws with suspension, creep and normal forces in which some of these forces are calculated by Lee et al. [9]. The vertical, roll, lateral and yaw equations of motion of single wheelset are

$$m_w \overset{\circ\circ}{Z}_{wi} + 2C_{pz}\overset{\circ}{Z}_{wi} - 2C_{pz}\overset{\circ}{Z}_{bj} - 2C_{pz}l_b\overset{\circ}{\theta}_{bj} + \frac{2f_{11}}{V}\lambda^2 Y_{wi}\overset{\circ}{\phi}_{wi}$$
$$+\frac{2f_{11}}{V}\phi_{wi}\overset{\circ}{Y}_{wi} + \frac{2f_{12}}{V}\phi_{wi}\overset{\circ}{\psi}_{wi} + \frac{2f_{11}r_o}{V}\phi_{wi}\overset{\circ}{\phi}_{wi} + 2K_{pz}Z_{wi}$$
$$-2K_{pz}Z_{bj} - 2K_{pz}l_b\theta_{bj} - \frac{2f_{12}}{r_o}\lambda^2 = 0 \tag{1}$$

$$J_{wx}\overset{\circ\circ}{\phi}_{wi} + \left(\frac{2f_{11}ar_o\lambda}{V} + \frac{2f_{11}r_o{}^2}{V}\right)\overset{\circ}{\phi}_{wi} + \frac{2f_{11}(r_o + a\lambda)}{V}\overset{\circ}{Y}_{wi}$$
$$-\left(\frac{J_{wy}V}{r_o} - \frac{2f_{12}r_o}{V} - \frac{2f_{12}a\lambda}{V}\right)\overset{\circ}{\psi}_{wi} + 2C_{pz}l_s\overset{\circ}{Z}_{wi} - 2C_{pz}l_s\overset{\circ}{Z}_{bj} - 2C_{pz}l_s l_b\overset{\circ}{\theta}_{bj}$$
$$-\left(2\lambda^2 f_{12} + a\lambda W\right)\phi_{wi} - \left(\frac{2f_{12}\lambda^2}{r_o}\lambda^2 W\right)Y_{wi} - \left(2f_{11}(r_o + a\lambda) + \frac{2f_{22}\lambda^2}{r_o}\right)\psi_{wi}$$
$$+2K_{pz}l_sZ_{wi} - 2K_{pz}l_sZ_{bj} - 2K_{pz}l_sl_b\theta_{bj} = 0 \tag{2}$$

$$m_w\overset{\circ\circ}{Y}_{wi} + \left(\frac{2f_{11}}{V} - 2C_{py}\right)\overset{\circ}{Y}_{wi} + 2C_{py}\overset{\circ}{Y}_{bj} + \frac{2f_{12}}{V}\overset{\circ}{\psi}_{wi} + \frac{2r_of_{11}}{V}\overset{\circ}{\phi}_{wi}$$
$$-2K_{py}Y_{wi} + 2K_{py}Y_{bj} - 2f_{11}\psi_{wi} + W\phi_{wj} = 0 \tag{3}$$

$$J_{wz}\overset{\circ\circ}{\psi}_{wi}+\left[\left(\frac{2a^2 f_{33}}{V}+\frac{2f_{22}}{V}\right)+2C_{px}L_x^{\ 2}\right]\overset{\circ}{\psi}_{wi}-\frac{2f_{12}}{V}\overset{\circ}{Y}_{wi}-\left(-\frac{J_{wy}V}{r_o}+\frac{2r_o f_{12}}{V}\right)\overset{\circ}{\phi}_{wi}$$
$$-2C_{px}L_x^{\ 2}\overset{\circ}{\psi}_{bj}+\left[2K_{px}L_x^{\ 2}-\left(-2f_{12}+a\lambda W\right)\right]\psi_{wi}-\frac{2a\lambda f_{33}}{r_o}Y_{wi}-2K_{px}L_x^{\ 2}\psi_{bj}=0$$
(4)

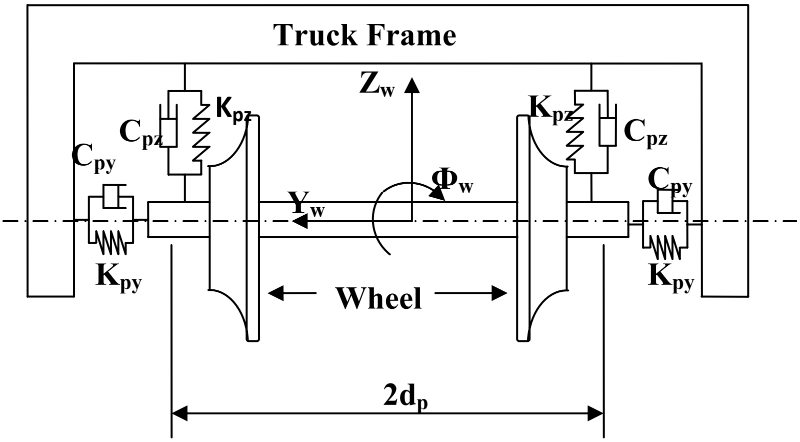

Figure (2) Front view of railway single wheelset equipped with sets of longitudinal, lateral and vertical primary suspension elements.

2.2 Bogies Differential Equations of Motion

Railway carriage model consists of two bogies in which each bogie has two conventional unconnected front and rear wheelsets and two vertical secondary suspension elements are used to connect bogies with carbody as shown in **Figure 3**. In additional to the set of primary suspension elements connected each bogie with the wheelsets. The bogies differential equations of motion govern vertical, pitch, roll, lateral, and yaw degrees of freedom are:

$$m_b\overset{\circ\circ}{Z}_{bj}+\left(2C_{sz}+4C_{pz}\right)\overset{\circ}{Z}_{bj}-2C_{sz}l_c\overset{\circ}{\theta}_c-2C_{sz}\overset{\circ}{Z}_c-2C_{pz}\overset{\circ}{Z}_{wi}-2C_{pz}\overset{\circ}{Z}_{wi}$$
$$+\left(2K_{sz}+4K_{pz}\right)Z_{bi}-2K_{sz}l_c\theta_c-2K_{sz}Z_c-2K_{pz}Z_{wi}-2K_{pz}Z_{wi}=0 \quad \textbf{(5)}$$

$$J_{by}\overset{\circ\circ}{\theta}_{bj}+4C_{pz}l_b^{\,2}\overset{\circ}{\theta}_{bj}-2C_{pz}l_b\overset{\circ}{Z}_{wi}+2C_{pz}l_b\overset{\circ}{Z}_{wi}+4K_{pz}l_b^{\,2}\theta_{bj}-2K_{pz}l_bZ_{wi}$$
$$+2K_{pz}l_bZ_{wi}=0 \qquad \textbf{(6)}$$

$$J_{bx}\overset{\circ\circ}{\phi}_{bj}+\left(4C_{pz}+2C_{sz}\right)l_s^{\,2}\overset{\circ}{\phi}_{bj}-2C_{sz}l_s^{\,2}\overset{\circ}{\phi}_{c}-2C_{pz}l_s^{\,2}\overset{\circ}{\phi}_{wi}-2C_{pz}l_s^{\,2}\overset{\circ}{\phi}_{wi}-2C_{sy}L_{cs}l_c\overset{\circ}{\psi}_{c}$$
$$+4C_{py}L_{cs}\overset{\circ}{Y}_{bj}-2C_{py}L_{cs}\overset{\circ}{Y}_{wi}-2C_{py}L_{cs}\overset{\circ}{Y}_{wi}+\left(4K_{pz}+2K_{sz}\right)l_s^{\,2}\phi_{bj}-2K_{sz}l_s^{\,2}\phi_c-2K_{pz}l_s^{\,2}\phi_{wi}$$
$$-2K_{pz}l_s^{\,2}\phi_{wi}-2K_{sy}h_{cs}l_c\psi_c+4K_{py}L_{cs}Y_{bj}-2K_{py}L_{cs}Y_{wi}-2K_{py}L_{cs}Y_{wi}=0$$

(7)

$$m_b\overset{\circ\circ}{Y}_{bj}+\left(2C_{sy}+4C_{py}\right)\overset{\circ}{Y}_{bj}-2C_{sy}\overset{\circ}{Y}_{c}-2C_{py}\overset{\circ}{Y}_{wi}-2C_{py}\overset{\circ}{Y}_{wi}+\left(2K_{sy}+4K_{py}\right)Y_{bj}-2K_{sy}Y_c$$
$$-2K_{py}Y_{wi}-2K_{py}Y_{wi}=0$$

(8)

$$J_{bz}\overset{\circ\circ}{\psi}_{bj}-\left(-4L_x^{\,2}C_{px}-4L_b^{\,2}C_{py}\right)\overset{\circ}{\psi}_{bj}-2L_x^{\,2}C_{px}\overset{\circ}{\psi}_{wi}-2L_x^{\,2}C_{px}\overset{\circ}{\psi}_{wi}-2L_bC_{py}\overset{\circ}{Y}_{w1}$$
$$+2L_bC_{py}\overset{\circ}{Y}_{wi}-\left(-4L_x^{\,2}K_{px}-4L_b^{\,2}K_{py}\right)\psi_{bj}-2L_x^{\,2}K_{px}\psi_{wi}-2L_x^{\,2}K_{px}\psi_{wi}-2L_bK_{py}Y_{wi}$$
$$+2bK_{py}Y_{wi}=0$$

(9)

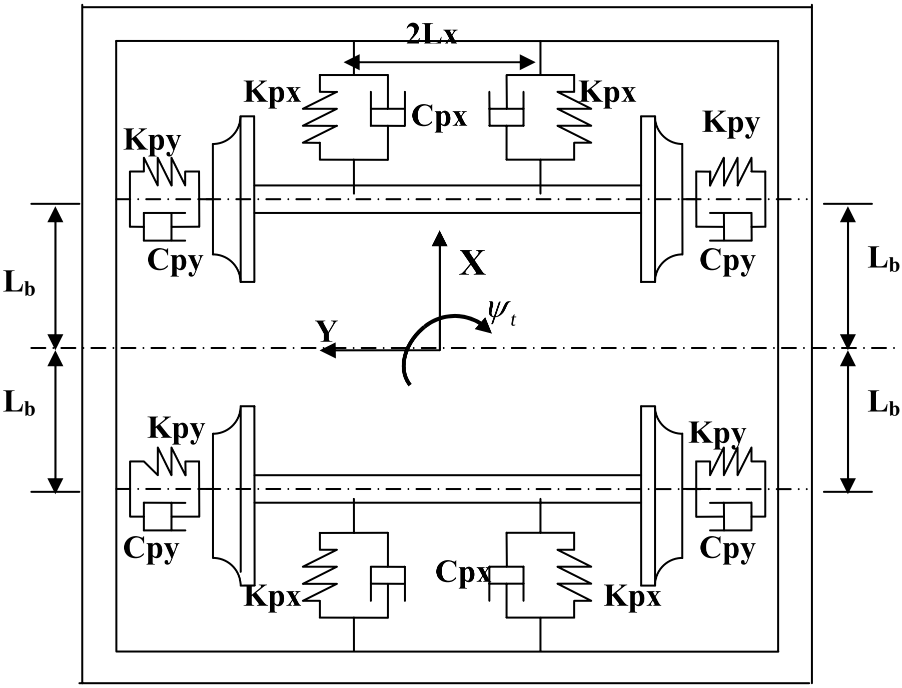

Figure (3) Top view of railway Bogie equipped with sets of longitudinal, lateral and vertical primary and secondary suspension elements.

2.3 Carbody Differential Equations of Motion

Carbody is the heaviest component in railway carriage makes crush between wheel and track and elastic deformation is introduced at contact patch area to produce creep forces and moments. Carbody differential equations of motion govern bounce, pitch, roll, lateral, and yaw degrees of freedom are derived applying Newton's law. The derived equations of motion of carbody with mass m_c and moment of inertia about longitudinal axis J_{cx}, about lateral axis J_{cy}, and about bounce axis J_{cz} are

$$m_c \overset{\circ\circ}{Z}_c + 4C_{sz} \overset{\circ}{Z}_c - 2C_{sz} \overset{\circ}{Z}_{bj} - 2C_{sz} \overset{\circ}{Z}_{bj} + 4K_{sz} Z_c - 2K_{sz} Z_{bj}$$
$$- 2K_{sz} Z_{bj} = 0$$

(10)

$$J_{cy}\ddot{\theta}_c + 4C_{sz}l_c^2\dot{\theta}_c - 2C_{sz}l_c\dot{Z}_{bj} + 2C_{sz}l_c\dot{Z}_{bj} + 4K_{sz}l_c^2\theta_c - 2K_{sz}l_cZ_{bj}$$
$$+ 2K_{sz}l_cZ_{bj} = 0$$

(11)

$$J_{cx}\ddot{\phi}_c + 4C_{sz}l_s^2\dot{\phi}_c - 2C_{sz}l_s^2\dot{\phi}_{bj} - 2C_{sz}l_s^2\dot{\phi}_{bj} + 4C_{sy}L_{cs}\dot{Y}_c - 2C_{sy}L_{cs}\dot{Y}_{bj} - 2C_{sy}L_{cs}\dot{Y}_{bj}$$
$$+ 4K_{sz}l_s^2\phi_c - 2K_{sz}l_s^2\phi_{bj} - 2K_{sz}l_s^2\phi_{bj} + 4K_{sy}L_{cs}Y_c - 2K_{sy}L_{cs}Y_{bj} - 2K_{sy}L_{cs}Y_{bj} = 0$$

(12)

$$m_c\ddot{Y}_c + 4C_{sy}\dot{Y}_c - 2C_{sy}\dot{Y}_{bj} - 2C_{sy}\dot{Y}_{bj} + 4L_{cs}C_{sy}\dot{\phi}_c + 4K_{sy}Y_c - 2K_{sy}Y_{bj} - 2K_{sy}Y_{bj}$$
$$+ \left(m_cg + 4L_{cs}K_{sy}\right)\phi_c = 0$$

(13)

$$J_{cz}\ddot{\psi}_c + 2\left(2l_c^2C_{sy} + C_{sx}\right)\dot{\psi}_c - C_{sx}\dot{\psi}_{bj} - C_{sx}\dot{\psi}_{bj} - 2l_cC_{sy}\dot{Y}_{bj} + 2l_cC_{sy}\dot{Y}_{bj}$$
$$+ 2\left(2l_c^2K_{sy} + K_{sx}\right)\psi_c - K_{sx}\psi_{bj} - K_{sx}\psi_{bj} - 2l_cK_{sy}Y_{bj} + 2l_cK_{sy}Y_{bj} = 0$$

(14)

3. Numerical Simulation

Railway carriage runs on tangent tracks is modeled by the second order differential equations of motion (1-14). A simple and important technique used to transform the governing equations of motion into first order differential equations in suitable form known as state space equations. This technique is used to facilitate solving the equations with numerical integration methods. The transformed equations of motion are simulated with computer-aided simulation to be solved by Runge-Kutta fourth order numerical method. The initial conditions of railway displacements are assumed as in [11] to obtain time domain solution at each step time input using Matlab Program. Data chosen to be applied in the numerical simulation model is adapted from resources [9, 11] also initial conditions are assumed for the dynamic motions of the system. Simulation is executed to represent the dynamic responses of railway carriage bogie subjected to different magnitudes of vertical primary suspension stiffness. Procedure is achieved by increasing the speeds to reach the critical hunting velocity of the model in which principles of phase plane approach are utilized to represent the critical hunting velocity of the system.

4. Conclusion

Numerical simulation model is constructed and used to study the dynamic behavior of railway bogie which is a component of railway carriage at critical hunting velocity due to different magnitudes of primary vertical suspension stiffness. The simulation results represent the dynamic behavior of a railway carriage bogie due to lateral, yaw, roll vertical and pitch dynamic responses with different magnitudes of primary vertical suspension stiffness at critical hunting velocity and subjected to selected magnitudes of wheel conicities. Roll, pitch and vertical dynamic behavior of railway bogie have distinct dynamic responses to change in magnitudes of primary vertical suspension stiffness at critical hunting velocity as shown in **Figures 6-8** meanwhile there is insensible change at lateral and yaw dynamic responses of railway bogie as shown in **Figures 4-5**. Lateral dynamic response of railway bogie is under the critical hunting velocity for different magnitudes of primary vertical suspension stiffness as depicted in **Figure 4**. Meanwhile yaw dynamic response of railway carriage bogie reaches the critical hunting velocity for different magnitudes of primary vertical suspension stiffness as shown in **Figure 5**. In which means that the railway carriage simulation model presents acceptable magnitudes of yaw and lateral displacements for railway bogie at different magnitudes of primary vertical suspension stiffness also lateral and yaw dynamic responses are not sensitive to change in magnitudes of vertical primary suspension stiffness but yaw response is more sensitive to critical hunting velocity of the system. **Figures 6-8** show acceptable magnitudes of roll, pitch and vertical dynamic response of railway bogie subjected to different magnitudes of vertical primary suspension stiffness represented by the constructed simulation model. The dynamic behavior of bogie due to these displacements show that high magnitudes of roll, pitch and vertical responses to low magnitudes of subjected vertical primary suspension stiffness and high magnitudes of vertical suspension stiffness give desirable low magnitudes of these displacements but use too high magnitudes of suspension stiffness means that the suspension will be rigid and so more undesirable vibrations are introduced. So that the present constructed simulation model should be used to select the appropriate suspension parameters. It is noticed that there is no influence with lateral and yaw dynamic behavior of railway bogie due to change in magnitudes of vertical primary suspension stiffness but there is distinct influence with roll, yaw and vertical dynamic responses. In additional roll, pitch and vertical dynamic responses are insensitive to critical hunting velocity unlike lateral and yaw in which are more sensitive to critical hunting velocity. It is concluded that change in magnitudes of vertical primary suspension stiffness has less influence to critical hunting velocity but the model is used to examine the appropriate magnitudes should be used for vertical primary suspension stiffness. Also it can be shown that yaw displacement is more sensitive to critical hunting velocity in which the yaw suspension parameters should be handled to increase the critical hunting velocity and improve the hunting phenomenon of railway carriage.

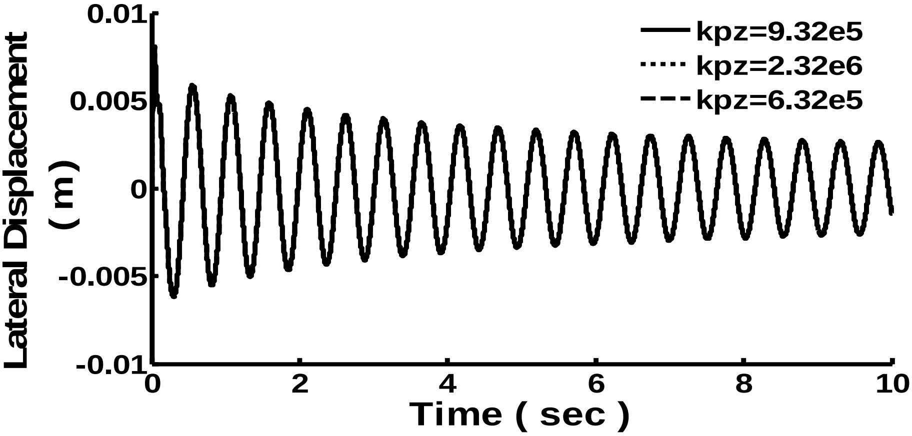

Figure (4) Lateral dynamic displacements of railway carriage Bogie at critical hunting velocity (99 km/h)

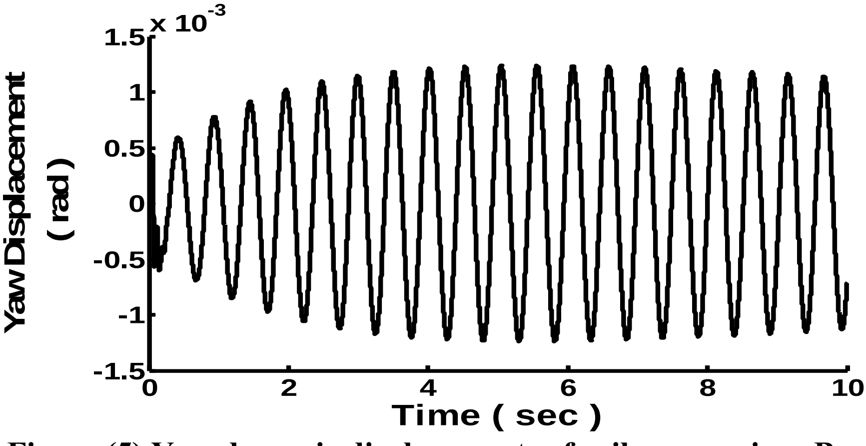

Figure (5) Yaw dynamic displacements of railway carriage Bogie at critical hunting velocity (99 km/h)

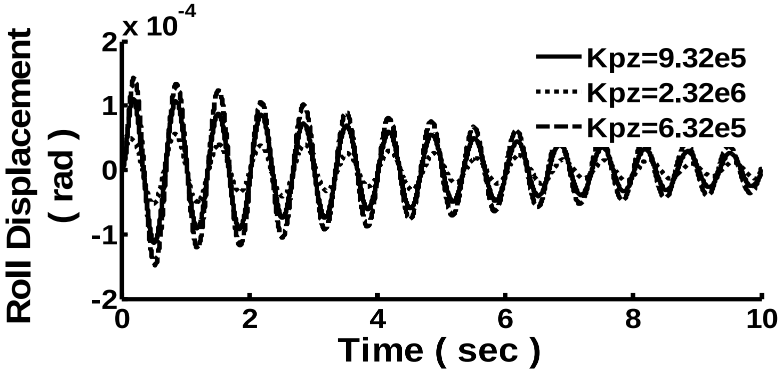

Figure (6) Roll dynamic displacements of railway carriage Bogie at critical hunting velocity (99 km/h)

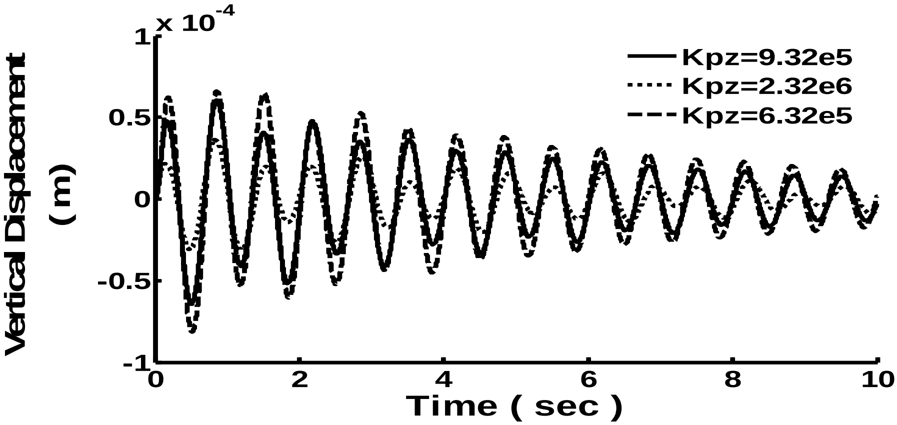

Figure (7) Vertical dynamic displacements of railway carriage Bogie at critical hunting velocity (99 km/h)

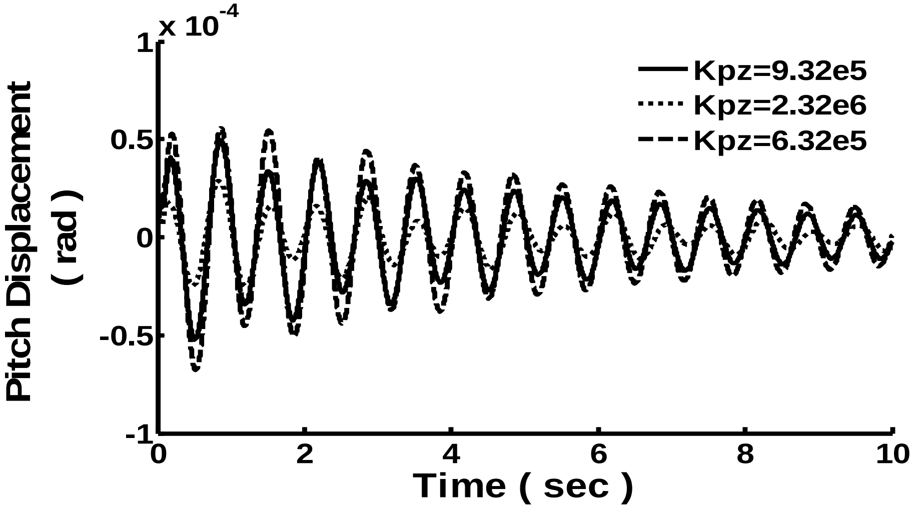

Figure (8) Pitch dynamic displacements of railway carriage Bogie at critical hunting velocity (99 km/h)

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