A New Method for Vertical Deformation Detection Using Survey Control Networks

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Abstract

Usually, the Survey control networks are used for deformation detection in a specific area using observations taken at different epochs. Where the coordinates obtained from two epochs, using least squares technique, are compared in order to assess if a deformation of a specified magnitude exists. Traditionally, the global congruency test is carried out so as to detect if the area of the network has undergone any movement (uplift or subsidence) due to natural or manmade causes. As a next step, localization methods are used to determine deformations at specific points in case there are changes in shape.

In this research a new method is developed to establish deformations at specific points directly. The method is tested using a vertical control network simulated at various epochs of observation. The results obtained are compared with those obtained by another used method. Results obtained using this method indicate that vertical deformations greater than 0.03m using a precision of observation less than $10\sqrt{k}$ can be detected at a minimum significant level of 0.05 (95% confidence level).

ملخص

عادة ما تستخدم شبكات الضبط في المساحة كواحدة من تقنيات إكتشاف الإزاحة لمساحة محددة علي سطح الأرض. باستخدام القياسات المأخوذة علي فترات زمنية مختلفة، ثم تتم مقارنة الإحداثيات المحسوبة عن طريق نظرية أقل التربيعات لتحديد مقدار الإزاحة أو الحركة لنقاط الشبكة الأرضية. جرت العادة علي استخدام الاختبار الشامل (global congruency test) لجميع نقاط الشبكة لتحديد ما اذا كان هنالك إزاحة كلية لمواقع نقاط الشبكة كنتاج للعوامل الطبيعية أو الحضارية. يتبع هذا الاختبار بأحد الاختبارات المحلية لتحديد مقدار الإزاحة عند كل نقطة علي حدة في حالة وجود تغيرات في الشكل. في هذا البحث تم تقديم طريقة حديثة لتحديد مقدار الإزاحة مباشرة للنقاط بصفة فردية. تم تطبيق هذه الطريقة علي شبكة راسية (leveling) صممت أرصادها علي فترات زمنية مختلفة ثم قورنت النتائج التي تم الحصول عليها مع واحدة من النظريات المستخدمة سلفاً. بعد تطبيق هذه الطريقة علي الشبكة موضوع الاختبار

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وجد أن الحد الأدنى للإزاحة الرأسية المكتشفة، مع استخدام $10\sqrt{k}$ كحد أقصى لدقة الرصد، حوالي ثلاثة سنتيمترات (0.030 متر) عندما تستخدم نسبة احتمال لوقوع خطأ من النوع الأول (α) لا تقل عن 5%.

1. Introduction

From the main aims of geodesy is deformation monitoring for an object or an area which is characterized with points of a geodetic network. It is essential to detect deformations for many purposes, for examples, monitoring plate tectonics or taking precaution for construction which may be under damage.

Conventionally, for all types of deformation networks displacements for some points are estimated using coordinates obtained from least squares adjustment of observations made at different epochs. Then, these displacements are tested using statistical methods to see if a significant movement exists. Therefore, this procedure is called as conventional or geometrical analysis which comprises global congruency test and localization methods steps. It can be, also, used to verify stabilities of some points or instability of others to define the reference points.

In the conventional deformation analysis (CDA) the deformations estimated using least squares technique are accepted as significant geometrical differences verified with a global statistical test. So, detection of the deformations basically depends on the comparison of geometry of the geodetic networks observed at different epochs.

The problem arises, that the global tests cannot reflect the poor part of the networks unless it is followed by one of the localization methods to detect the points have largest deformation size. From here the idea of presenting a more simple and direct method is arisen.

2. Adjustment of the vertical Deformation networks:

When the reference points of the network are defined, as in deformation monitoring in engineering structures case, the adjustment can be obtained using these fixed points to define the datum i.e. using fixed datum. Otherwise, i.e. no datum can be defined the heights and their variances for each epoch are obtained using the free datum adjustment. In this case all heights of the network are considered in the model as unknowns. So the Gauss-Markov model can be expressed as (Cross, 1983, Chen, 1983):

$$(Ax = \ell + v)_i$$

$$(W = C_\ell^{-1})_i$$

where:

i : is the number of epochs, A : is the $(n \times m)$ design matrix, x : is the $(m \times 1)$ unknowns vector, ℓ : is the $(n \times 1)$ observations vector, v : is the $(n \times 1)$ residuals vector, W : is the $(n \times n)$ diagonal weight matrix, C_ℓ^{-1} : is the covariance matrix of observations and n, m : are the number of observations and the number of unknowns respectively.

When the conditions $v^T W v = \min$, $x^T x = \min$ are satisfied, estimated unknown vector can be formulated as:

$$\hat{x}_i = C_x^+ A^T W \ell_i$$

where C_x^+ is the covariance matrix of estimated unknowns and obtained from (Grafarend. and Schattrin, 1973):

$$C_x^+ = (A^T W A)^+ = N^+$$

where $(+)$ means the pseudo inverse and obtained in the case of the free networks adjustment as follows (Caspary, 1987):

$$C_x^+ = N^+ = (N + S S^T)^{-1} S (S^T S)^{-1} S^T$$

where S is the matrix express datum defect and it will be $(m \times 1)$ unit vector in the case of the vertical networks.

3. Global Congruency Test

The global congruency test is to detect whether or not the points group considered has remained stable. It is applied to see if the differences between the estimated coordinates of the common points are the result of the deformation or only random errors. The null hypothesis of the global congruency test is read (Hekimogolu, Demiret and Aydin, 2002):

H_0 : the common points of both epochs are stable and thus have the same expectation of the estimated positions.

i.e. $H_o : E(\hat{x}_1) = E(\hat{x}_2) = x$

or $H_o : E(d\hat{x}) = 0$

The influence of this hypothesis can be expressed by the following quadratic form:

$$\Omega = d\hat{x}^T W_{d\hat{x}} d\hat{x}$$

where:

$d\hat{x}$: is the deformation vector obtained from the estimated heights of two epochs.

$W_{d\hat{x}}$: is the weight matrix of the deformation vector obtained by inversion of the deformation covariance matrix as follows:

$$C_{d\hat{x}} = C_{\hat{x}_1} + C_{\hat{x}_2}$$

$$W_{d\hat{x}} = C_{d\hat{x}}^{-1}$$

The test statistic is established as follows (Cooper, 1987):

$$\omega = \frac{\Omega}{h\sigma_o^2} \sim F_{h,f}$$

where:

h : is the rank of the covariance matrix of the deformation vector.

$\hat{\sigma}_o^2 = (f_1 \hat{\sigma}_{o1}^2 + f_2 \hat{\sigma}_{o2}^2) / f$: is the unit weight variance for the two epochs, in which:

$\hat{\sigma}_{o1}^2$ and $\hat{\sigma}_{o2}^2$: are the unit weight variances of the two epochs.

$f = f_1 + f_2$: are the degrees of freedoms of combined two epochs, first epoch and second epoch respectively.

The significance level for the test is selected. Then, if the computed value of ω is less than or equal to tabulated one $F_{h,f,\alpha}$, then the null hypotheses is accepted and the network at the second epoch is assumed to be congruent with that at the first epoch, i.e. the shapes are the same.

The validity of the global test depends upon the observations at each epoch being free from systematic and gross errors. Therefore, before making the test, it is necessary to apply one of the methods of gross error detection and systematic error to be corrected. If the null hypotheses of the global congruency test is rejected i.e. there have been a significant deformable point(s), then the next step is, usually, the identification of any part of the network which has retained stable. Therefore, one of the following localization methods is required to be used.

4. Localization Methods of Deformation Detection

The main objective of the localization methods is to identify the group of points are significantly stable; hence they can be used for the datum definition. This is done by applying individual tests or single point tests to identify the point have the greatest influence of the quadratic form. Then this point should be eliminated from the group of the points define the datum. Many localization methods are presented. The following are examples of some methods to be used.

4.1. Localization with Gauss Elimination

The rejection of the global concurrency test means that there is a deformed point at least. To identify this point the deformation vector and its covariance matrix are divided into sub-vectors and sub-matrices as follows (Caspar, 1987):

$$d\hat{x} = \begin{bmatrix} d\hat{x}_r \\ d\hat{x}_p \end{bmatrix}$$

$$W_{d\hat{x}} = C_{d\hat{x}}^+ = \begin{bmatrix} W_{rr} & W_{rp} \\ W_{pr} & W_{pp} \end{bmatrix}$$

where

index p describes assumed deformed point P and index r for the other reference points.

- To distinguish independently the effect of the point P on the quadratic form of the congruency test, the following equation is obtained by using the Gauss elimination method.

$$d'\hat{x}_p = d\hat{x}_p + W_{pp}^{-1}W_{pr}d\hat{x}_r$$

The quadratic form for point P is obtained as follows:

$$\Omega_p = d'\hat{x}_p^T W_{pp} d'\hat{x}_p$$

- Sequentially, each point is considered as deformed point. The point which gives maximum quadratic form is accepted as deformed point and eliminated from set of the reference points.

- To test whether remaining points are deformed, the new deformation vector and its covariance matrix are transformed into the current datum with using transformation model in chapter two as follows:

$$d_i \hat{x} = k_i d\hat{x}$$

$$C_{d_i \hat{x}} = k_i C_{d\hat{x}} k_i^T$$

where k is the transformation matrix and given by (Caspary, 1987) as :

$$k = I - S(D^T S)^{-1} D^T$$

where I is (mxm) identity matrix, D is the (mxI) datum constraints vector with the unit elements for points defining the datum and zero for others and S is previously defined.

- Using results obtained from upper transformation a new global congruency test is applied.
- The same localization procedure is repeated until the test can not verify existence of deformation i.e. all deforms points are detected.
- For every point The test statistic :

$$T = \frac{\Omega_p}{\sigma_o^2}$$

can be computed and compared to tabulated value.

4.2. Localization with Implicit Hypotheses

The main idea of this method is the investigation of residuals obtained from combined adjustment of two epochs (Caspary, 1983).

Consideration the following combined adjustment model:

$$\begin{bmatrix} A_1^r & A_1^p & 0 & A_1^o & 0 \\ A_2^r & 0 & A_2^p & 0 & A_2^o \end{bmatrix} \begin{bmatrix} x^r \\ x_1^p \\ x_2^p \\ x_1^o \\ x_2^o \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

where

A_1^p and A_2^p : are the design matrices related to point P in two epochs.

x_1^p and x_2^p : are the coordinates to be estimated for point P in two epochs.

The point P is separated from the reference points (superscripted by r) to be adjusted separately using the observations of two epochs.

This means that the condition $\hat{x}_1^p = \hat{x}_2^p$ has been removed and leads to the reduced new quadratic form.

The procedure of the test statistic can be carried out (after failure of the congruency test) to explain the effect of the point P on the quadratic form under the null hypotheses:

$$E(\hat{x}_1^p) = E(\hat{x}_2^p)$$

- The following statistic quantity is computed for each eliminated point (P)

$$q_p = v^T W v - (v_1^T W_1 v_1 + v_2^T W_2 v_2)$$

where

$$W = \begin{bmatrix} W_1 & 0 \\ 0 & W_2 \end{bmatrix}$$

- The point P that gives minimum q_p is defined as the deformed point and removed from datum points to obtain a new congruency test.
- The procedure is repeated for the new failure congruency test after the point has minimum quadratic is removed from the reference points.

The test can be carried out using other procedure

- The following quadratic form is computed from the deformation vector and its covariance matrix (as in global test)

$$\Omega_p = \Omega - \Omega'$$

where

Ω_p : is the quadratic of any point P.

Ω : is the quadratic form of reference points.

Ω' : is the quadratic form of reference points after the point P is removed.

- The test statistic :

$$T = \frac{\Omega_p}{\sigma_o^2} \prec F(1, \infty)$$

is calculated for each point .

- The point with the largest statistic is considered as unstable and transferred from the block of reference points.
- The procedure is repeated until the result of the concurrency test is accepted for remained points.

5. The new method

The deformation vector either reflects the results of only random errors in observations or reflects the existence of deformed point(s) between two epochs. Therefore, when the adjusted networks of two epochs are free of deformable points, then the deformation vector is normally distributed with zero mean and variances σ_{dx}^2 .

$$d\hat{x} \sim N(0, C_{d\hat{x}})$$

On the other hand, the expected value of the deformation vector is not equal to zero (shifted mean) in the case of the existence of deformed point(s) between two epochs. Hence the deformation vector is normally distributed with a shifted mean μ and variance σ_{dx}^2 .

$$d\hat{x} \sim N(\mu, C_{d\hat{x}})$$

From the null and alternative hypotheses:

$$H_o : E(d\hat{x}) = 0$$

$$H_A : E(d\hat{x}) \neq 0$$

The hypotheses test can be established for the elements of deformation vector individually as follows:

$$d\hat{x}_i \sim N(0, \sigma_{dx_i}^2)$$

with standard form

$$H_o : \frac{d\hat{x}_i}{\sigma_{d\hat{x}_i}} = \omega_i \sim N(0, 1)$$

Similarly:

$$H_A : \frac{d\hat{x}_i}{\sigma_{d\hat{x}_i}} = \omega_i \sim N(\mu, 1)$$

From which it is shown that the shift of mean from zero to μ is caused by existence of a deformable point(s).

- The ratio between the deformation and its standard error, which is denoted by ω_i in the upper two equations, can be computed for all elements of the deformation vector.
- The computed value of this ratio for every element is compared with the tabulated one obtained using specified level of significance.
- If the computed value is less than the tabulated one (accepted test result), then there is no significant movement in the corresponding point. But if the result of the test is rejected, this means that the corresponding point of this element is significantly moved and these points of rejected test are considered as deformable points.

6. Simulated Vertical Network

A leveling network consisting of seven stations (Fig-1) is simulated such that individual loop misclosures shall not exceed $10\sqrt{k}$ millimeters where k is the distance leveled in kilometers.

Simulated observations and their standard deviation in Table (1) are considered to be an initial epoch (epoch No.1).

Fig (1)-A Simulated Vertical Control Network

Table (1) - Simulated Observations of Epoch No.(1)

Observation No.	From Point	To point	Observation (m)	Distance (km)	Standard error (m)
1	1	2	1.510	2.000	0.014
2	1	3	2.775	2.500	0.016
3	1	4	1.618	2.549	0.016
4	2	3	1.250	2.061	0.014
5	3	4	-1.175	1.118	0.011
6	3	6	1.322	2.828	0.017
7	4	5	1.800	1.118	0.011
8	4	6	2.480	3.354	0.018
9	5	6	0.700	2.549	0.016
10	5	7	1.876	4.031	0.020
11	6	7	1.200	2.121	0.015

Another four epochs are generated by altering observations by multiples of their standard deviations (half, one, two and three times), respectively, while keeping the loop misclosures to acceptable limits as shown in Table (2).

Table (2) - Simulated Observations of the Four Epochs

Observation No.	Observations Of epoch(1) (m)	Observations Of epoch(2) (m)	Observations Of epoch(3) (m)	Observations Of epoch(4) (m)
1	1.510	1.503	1.496	1.468
2	2.775	2.767	2.759	2.727
3	1.618	1.610	1.602	1.570
4	1.250	1.249	1.248	1.244
5	-1.175	-1.175	-1.175	-1.175
6	1.322	1.314	1.305	1.271
7	1.800	1.795	1.789	1.767
8	2.480	2.472	2.463	2.429
9	0.700	0.697	0.694	0.682
10	1.876	1.866	1.856	1.816
11	1.200	1.193	1.186	1.158

7. Tests and Results

The first station (1) is assumed to be fixed with zero variance. Using the least squares method the heights of the points from 2 to 7 are estimated

for each epoch. The displacements (differences in estimated heights) and their standard deviations are shown in Table (3).

Table (3)-Displacements and Their Standard Deviations from the Four Epochs

Point No.	Displacement from epoch (2)(m)	Displacement from epoch (3)(m)	Displacement from epoch (4)(m)	Displacement from epoch (5)(m)	Standard deviation ($\sigma_{d\hat{x}_i}$)m
2	-0.007	-0.014	-0.028	-0.042	0.0158
3	-0.008	-0.016	-0.032	-0.048	0.0147
4	-0.008	-0.016	-0.032	-0.048	0.0159
5	-0.013	-0.027	-0.054	-0.081	0.0202
6	-0.016	-0.033	-0.066	-0.099	0.0202
7	-0.023	-0.047	-0.094	-0.141	0.0254

The global congruency test is applied at a 5% significant level to the displacements obtained from the four epochs and the results of the test are shown in Table (4).

Table (4)-Results of the Global Test for all Epochs

Epoch No.	$\varpi = \Omega / (h \cdot \sigma_o^2)$	Null hypothesis (H_0)
2	0.335	Accepted
3	1.381	Accepted
4	5.523	Rejected
5	12.427	Rejected

The new method for individual point testing is applied to the displacements from the four epochs shown in Table (3). The results are shown in Tables (4), (5), (6) and (7).

Table (4)-Results of Epoch (2)

Point No.	Displacement(m) $d\hat{x}_i$	Standard deviation (m) $\sigma_{d\hat{x}_i}$	Test statistic $\hat{\omega}_i = \left \frac{d\hat{x}_i}{\sigma_{d\hat{x}_i}} \right $	Null hypothesis (H_0)
2	-0.007	0.0158	0.443	accepted
3	-0.008	0.0147	0.646	accepted
4	-0.008	0.0159	0.503	accepted
5	-0.013	0.0202	0.645	accepted

6	-0.016	0.0202	0.793	accepted
7	-0.023	0.0254	0.907	accepted

Table (5)-Results of Epoch (3)

Point No.	Displacement(m) $d\hat{x}_i$	Standard deviation (m) $\sigma_{d\hat{x}_i}$	Test statistic $\hat{\omega}_i = \left \frac{d\hat{x}_i}{\sigma_{d\hat{x}_i}} \right $	Null hypothesis (H ₀)
2	-0.014	0.0158	0.887	accepted
3	-0.016	0.0147	1.091	accepted
4	-0.016	0.0159	1.006	accepted
5	-0.027	0.0202	1.339	accepted
6	-0.033	0.0202	1.635	accepted
7	-0.047	0.0254	1.853	accepted

Table (6)-Results of Epoch (4)

Point No.	Displacement(m) $d\hat{x}_i$	Standard deviation (m) $\sigma_{d\hat{x}_i}$	Test statistic $\hat{\omega}_i = \left \frac{d\hat{x}_i}{\sigma_{d\hat{x}_i}} \right $	Null hypothesis (H ₀)
2	-0.028	0.0158	1.772	accepted
3	-0.032	0.0147	2.183	rejected
4	-0.032	0.0159	2.012	rejected
5	-0.054	0.0202	2.678	rejected
6	-0.066	0.0202	3.270	rejected
7	-0.094	0.0254	3.706	rejected

Table(7)-Results of Epoch (5)

Point No.	Displacement(m) $d\hat{x}_i$	Standard deviation (m) $\sigma_{d\hat{x}_i}$	Test statistic $\hat{\omega}_i = \left \frac{d\hat{x}_i}{\sigma_{d\hat{x}_i}} \right $	Null hypothesis (H ₀)
2	-0.042	0.0158	2.658	rejected
3	-0.048	0.0147	3.274	rejected
4	-0.048	0.0159	3.018	rejected
5	-0.081	0.0202	4.107	rejected
6	-0.099	0.0202	4.905	rejected
7	-0.141	0.0254	5.559	rejected

8. Discussion

The global test cannot detect deformations from displacements obtained from epochs two and three at 5% significant level i.e. the summation of the squares of the weighted deformations are too small to be detected. On the other hand, the test detected deformations from displacements obtained from epochs four and five at the same significant level i.e. an overall deformation exists.

At 0.05 significant level the new method failed to detect point deformations from displacements of epochs two and three. However, referring to epoch four, the method was successful in detecting deformations from displacements at all stations except that of station two. For results obtained from epoch five all stations displacements were detected as deformations.

9. Conclusions

- The new method gives the same results obtained from the global concurrency test i.e. If the global congruency test fails to detect an overall deformation then the new method will not be able to detect individual point deformation and vice versa.
- The new method detects deformation at specific points directly. This means the procedure can be used to determine the most stable area for the required development.
- Changes in observations by more than twice of their standard deviations lead to a detected deformation at 5% or greater significant level.
- The new method can easily detect individual vertical deformations of size greater than three centimeters with standard error of fifteen millimeters at a 5% significant level.

10. References

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