

Some Related Theorems of Gauss Bonnet Theory

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Abstract

This study aims to explain Some Related Theorems of Gauss Bonnet theory. We used the historical analysis mathematical method. We found that the Gauss Bonnet theory relates the Gaussian curvature of a surface to the geodesic curvature, this theory contains many applications within and outside of its original field of differential geometry, and the Gauss Bonnet theory is one of the most important and one of the deepest result in the differential geometry of surfaces.

المستخلص

تهدف هذه الدراسة لتوضيح بعض المبرهنات المرتبطة بنظرية جاوس بونيه، استخدمنا المنهج التاريخي التحليلي الرياضي. وجدنا أن نظرية جاوس بونيه تربط الانحناء الجاوسي للسطح بالانحناء الجيوديسي، وتحتوي هذه النظرية على العديد من التطبيقات في مجالها الداخلي والخارجي للهندسة التفاضلية، كما أن نظرية جاوس بونيه هي واحدة من أهم وأدق النتائج في الهندسة التفاضلية للسطوح.

Key Wards: Related theorems , Gauss Bonnet Theory

1.Introduction:

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The Gauss-Bonnet theory relates the sum of the interior angles of a triangle with its Gaussian curvature, an intrinsic quantity of the geometry of the space that the triangle is drawn on. The theory has numerous applications within and without its native field of differential geometry. In order to understand the Gauss-Bonnet theory we must first understand some basic differential geometry. To this end we start with the most basic idea in differential geometry: a regular surface.

The Gauss-Bonnet theory bridges the gap between topology and differential geometry. Its importance lies in relating geometrical information of a surface to a purely topological characteristic, which has resulted in varied and powerful applications. Through this study, presented with no original mathematics, it carefully works through the necessary tools for proving Gauss-Bonnet. Gauss first proved this theory in 1827, for the case of a hyperbolic triangle. This theory established a remarkable invariant relating curvature to the notion of angle within the surface. However, with the developments in topology in the 19th and 20th centuries, this theory has become an invaluable piece of modern mathematics. [10]

2. Regular Surface:

Definition (2.1): A subset $S \subset \mathbb{R}^3$ is a regular surface if for every point $P \in S$ there is a neighborhood $V \subset \mathbb{R}^3$ and a function f which maps an open set $U \subset \mathbb{R}^2$ onto $V \cap S \subset \mathbb{R}^3$ which has the following properties:

f is differentiable, f is a homeomorphism and for every $q \in U$, the differential $df_q: \mathbb{R}^2 \rightarrow \mathbb{R}^3$ is one to one. [6]

Definition (2.2): Let S denote a regular surface and $P_0 \in S$.

- 1) The linear tangent $T_{P_0}S$ to S at P_0 consists of all velocity vectors to smooth curves on S through P_0 . Given a

coordinate patch $r: D \rightarrow R^3$ for S with $r(u_0, v_0) = \overline{OP_0}$, it has a parameterization.

$$Tp_0S = \{sru(u_0, v_0) + trv(u_0, v_0), s, t \in R\}$$

2) The affine tangent plane $\pi_{p_0}S$ to S at P_0 to S at P_0 consists of all points $Q \in E^3$ with $\overline{P_0Q} \in Tp_0S$. It has a parameterization

$$\pi_{P_0}S = \{Q \in E^3 \mid \overline{OQ} = \overline{OP_0} + sru(u_0, v_0) + trv((u_0, v_0)), s, t \in R\}. [16]$$

Example (2.3): Let $\alpha: I \rightarrow R^3$ be a regular parameterized curve define $x(t, v) = \alpha(t) + v\alpha'(t)$, $(t, v) \in I \times R$.

x is a parameterized surface called the tangent surface.

Assume now that the curvature $k(t)$, $t \in I$, of α is nonzero for all $t \in I$, and restrict the domain of x to $U = \{(t, v) \in I \times R, v \neq 0\}$

Then:

$$\frac{\partial x}{\partial t} = \alpha'(t) + v\alpha''(t), \quad \frac{\partial x}{\partial v} = \alpha'(t)$$

and

$$\frac{\partial x}{\partial t} \cap \frac{\partial x}{\partial v} = v\alpha''(t) \cap \alpha'(t) \neq 0 \quad (t, v) \in U$$

Since, for all, the curvature

$$k(t) = \frac{|\alpha''(t) \cap \alpha'(t)|}{|\alpha'(t)|^3}$$

is nonzero. It follows that the restriction $x: U \rightarrow R^3$ is a regular parameterized surface, the trace of which consists of two connected pieces whose common boundary is the set $\alpha(I)$. [6]

Theorem (2.4): Let $x: U \subset R^2 \rightarrow R^3$ be a regular parameterized surface and let $q \in U$. Then there exists an neighborhood v of q in R^2 such that $x(v) \subset R^3$ is a regular surface.

Proof:

This is a gain a consequence of the inverse function theory. Write

$$x(u, v) = (x(u, v), y(u, v), z(u, v)).$$

By regularity, we can assume that:

$(\partial(x, y) / \partial(u, v))(q) \neq 0$ define a map

$F: U \times \mathbb{R} \rightarrow \mathbb{R}^3$ by

$$F(u, v, t) = (x(u, v), y(u, v), z(u, v) + t), (u, v) \in U, t \in \mathbb{R}$$

Then:

$$\det(df_q) = \frac{\partial(x, y)}{\partial(u, v)}(q) \neq 0$$

by the inverse function theory, there exist neighborhoods w_1 of q and w_2 of $F(q)$ such that $F: w_1 \rightarrow w_2$ is a diffeomorphism set $V = w_1 \cap U$ and observe that the restriction $F|_V = x|_V$. Thus, $x(V)$ is diffeomorphic to V , and hence a regular surface. [6]

Example (2.5): Let $z = f(x, y)$ where f differentiable function is we define a coordinate patch as follows: $x(u, v) = (u, v, f(u, v))$ to see that x is indeed:

i) x is 1-1 map if $P_1 = (u_1, v_1), P_2 = (u_2, v_2)$ and $p_1 \neq p_2$ then

$$x(p_1) \neq x(p_2)$$

ii) x is differentiable since:

$$X_u \equiv \frac{\partial x}{\partial u}, X_v \equiv \frac{\partial x}{\partial v}$$

Path exist, infect:

$$X_u = (1, 0, f_u), X_v = (0, 1, f_v)$$

X_u and X_v are continuous. [8]

3. First and Second Fundamental forms:

i. First fundamental form: Tells how the surface inherits the natural inner product of \mathbb{R}^3 .

Definition (3.1): Let M be a regular surface in \mathbb{R}^3 and let $P \in M$ the first fundamental form of M at p is the map:

$$I_p = I_p M \times T_p M \rightarrow \mathbb{R}$$

$$(x, y) \rightarrow \langle x, y \rangle. (2.3)$$

We will, for convenience, use the notation $f_x = \frac{\partial f}{\partial x}$ for the partial derivative of a function f with respect to a variable x . we want to write the first fundamental form in terms of the basis associated with the local chart (ϕ, U) . Remember that an element of $T_p M$ is a tangent vector at a point $p = \gamma(0) \in M$ to parameterized curve $\gamma(t) = \phi(u(t), v(t)), t \in (-\varepsilon, \varepsilon)$.

It holds that:

$$\begin{aligned} Ip(\gamma', \gamma') &= \langle \phi_u \cdot \frac{du}{dt} + \phi_v \cdot \frac{dv}{dt}, \phi_u \cdot \frac{du}{dt} + \phi_v \cdot \frac{dv}{dt} \rangle = \\ &= \langle \phi_u, \phi_u \rangle \left(\frac{du}{dt}\right)^2 + 2\langle \phi_u, \phi_v \rangle \frac{du}{dt} \frac{dv}{dt} + \langle \phi_v, \phi_v \rangle \left(\frac{dv}{dt}\right)^2 \\ &= E \left(\frac{du}{dt}\right)^2 + 2F \frac{du}{dt} \frac{dv}{dt} + G \left(\frac{dv}{dt}\right)^2, \end{aligned}$$

Where we define:

$$E = \langle \phi_u, \phi_u \rangle, F = \langle \phi_u, \phi_v \rangle, G = \langle \phi_v, \phi_v \rangle$$

The first fundamental form is often written in the modern notation of the metric tensor:

$$(g_{ij}) = \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix} = \begin{pmatrix} E & F \\ F & G \end{pmatrix}. [9]$$

Example (3.2): For the plane parameterized by $\sigma(u, v) = p + uq_1 + vq_2$, where q_1, q_2 are linearly independent vectors in \mathbb{R}^3 , we have $\sigma'_u = q_1$ and $\sigma'_v = q_2$. It follows that the component function are constant:

$$E = \|q_1\|^2, F = q_1 \cdot q_2, G = \|q_2\|^2.$$

In particular, if q_1, q_2 is an orthonormal pair, we have $E = G = 1, F = 0$. [11]

Definition (3.3): By restricting the natural inner product $\langle \cdot, \cdot \rangle$ on \mathbb{R}^3 to each tangent plane $T_p(S)$ of a regular surface S , We get

an inner product on $T_p(S)$. we call this inner product on T_pS the first fundamental form denote it by I_p . So , $I_p(u, w) = \langle u, w \rangle$. [1]

ii.Second fundamental form:

Definition (3.4): The second fundamental form of a regular oriented surface M at a point $P \in M$ is the map:

$$II_p = T_pM \times T_pM \rightarrow R$$

$$(x, y) \rightarrow \langle L_p(x), y \rangle. [1]$$

Theory (3.5): The normal curvature in direction T_o , is

$$k_n = \frac{II_p(w_o)}{\|w_o\|^2}$$

Proof:

Let $\gamma = \sigma_o \mu$ be curve on σ with $\mu(t_o) = p$ and $\gamma'(t_o) = w_o$,
 $= -\gamma'(t_o) \cdot m'(t_o) / \|\gamma'(t_o)\|^2$.

Hence $kn = w_o \cdot w_p(w_o) / \|w_o\|^2$.

Assume $\|w_o\| = 1$, then $II_p(w_o)$ is normal curvature at p of any curve on σ , which has the tangent vector w_o in this point. The relation describes the geometric content of the second fundamental form.[11]

Definition (3.6): A coordinate patch $x: D \rightarrow R^3$ is a one-to-one regular mapping of an open set D of R^2 into R^3 . [3]

Definition (3.7): Differentiable curve $\alpha : [a, b] \rightarrow IR^3$ is parameterized by arclength

$$\|\alpha'(t)\| = 1 \text{ for } t \in [a, b]$$

Curvature measures how fast the tangent vector rotates torsion characterizes the non-planarity of a three- dimensional space curve. [4]

Definition (3.8): The function that measures the angle of rotation for the unit normal vector, or equivalently, the unit tangent vector. In terms of the gauss map, the head of the unit normal vector always lies on the unit circle therefore, the

derivative of the unit normal vector must always be tangent to the unit circle. This is a manifestation of the fact that the derivative of a vector function that has constant magnitude is always perpendicular to the original vector function. Two notions point the way. First, over small distances, the arc of a circle near a point on the circle and the tangent line through that point are very similar. Second, the length of an arc of the unit circle is equal to the corresponding angle measured in radians. Therefore, a derivative of the unit normal vector measures change along a tangent to the unit circle (as in the Gauss map), this change is essentially the same as the change along the unit circle, which is equal to a change in the direction of the normal vector measured in radians. In other words, the conclusions of the last section suggest that the curvature can be defined as the derivative of the unit normal vector with respect to arc length. It can also be defined as the derivative of the unit tangent vector with respect to arc length.[7]

That is,

$$k(s) = \left\| \frac{dn}{ds} \right\| = \left\| \frac{dt}{ds} \right\|$$

Definition (3.9): Let p be a point of $M \subset R^3$. The maximum and minimum values of normal curvature $k(u)$ of M at p are called the principal curvatures of M at p , and are denoted by k_1 and k_2 . The directions in which these extreme values occur are called principal directions of M at p . unit vectors in these directions are called principal vectors of M at p . [3]

Example (3.10): Find the principal curvature and principal direction for Shape operator $s_p(e_1) = -\frac{1}{r}$

Solution

$$s_p(e_2) = 0e_2$$

The principal curvatures are: $-\frac{1}{r}, 0$ and the principal direction are: e_1, e_2 . [2]

4. Sectional Curvature:

Definition (4.1): Let $M \subset \mathbb{R}^m$ be a smooth m -dimensional manifold. Let $p \in M$ and $E \subset T_p M$ be a 2 – dimensional linear subspace of the tangent space. The sectional curvature of M at (p, E) is the number

$$k(p, E) = \frac{\langle R_p(u, v)u, v \rangle}{|u|^2|v|^2 - \langle u, v \rangle^2}$$

Where $u, v \in E$. [13]

Example (4.2): If $M \subset \mathbb{R}^3$ is a 2-manifold, the sectional curvature $k(p, T_p M) = k(p)$ is the Gaussian curvature of M at p . more generally, for any 2-manifold $M \subset \mathbb{R}^m$ (whether or not it has codimension one) we define the Gaussian curvature of M at p by:

$$K(p) := k(p, T_p M)$$

Example (4.3): If $M \subset \mathbb{R}^{m+1}$ is a sub-manifold of co-dimension one and

$v: M \rightarrow S^m$ is a Gauss map then the sectional curvature of a 2 – dimensional subspace $E \subset T_p M$ spanned by two linearly independent tangent vectors $u, v \in T_p M$ is given by:

$$k(p, E) = \frac{\langle u, dv(p)u \rangle \langle u, dv(p)v \rangle - \langle u, dv(p)v \rangle^2}{|u|^2|v|^2 - \langle u, v \rangle^2}$$

Where $M = S^m$, we have $v(p) = p$ and hence $k(p, E) = 1$ for all p and E . for a sphere of radius r we have $v(p) = p/r$ and hence $k(p, E) = 1/r^2$. [13]

Definition (4.4): Let $t \in I$ and assume that $k(t) \neq 0$

The number:

$$T(t) = \det \frac{[\gamma'(t)\gamma''(t)\gamma'''(t)]}{\|\gamma'(t) \times \gamma''(t)\|^2}$$

is called the torsion of γ at t . [13]

5. Gauss – Bonnet Formula: Relates the Gaussian Curvature of a surface to the geodesic curvature of a curve and leads to the Gauss- Bonnet theory. [9]

Definition (5.1): Let γ be a regular curve on the regular surface M . The geodesic curvature of γ at a given point $p \in \gamma$ is defined as

$$\kappa_{\gamma}(p) := \langle \ddot{\gamma}, N \times \dot{\gamma} \rangle. (4.1) [9]$$

Definition (5.2): A triangulation of a compact regular surface M consists of a finite family of closed subsets $\{T_1, \dots, T_n\}$ and homeomorphisms $\{\varphi_i: T' \rightarrow T_i \in \mathbb{R}^2\}$ where T is a triangle in \mathbb{R}^2 such that:

$\cup_i T_i = M$ and for $i \neq j, T_i \cap T_j \neq \emptyset$ implies $T_i \cap T_j$ is either a single vertex or a single edge. [9]

Remark (5.3): We will now prove the local case of Gauss-Bonnet. However, in order to do this, we first need to define some key resources from differential geometry. These notions of curvature tell us roughly what a surface looks like both locally and globally. [9]

Definition (5.4): The Gaussian curvature κ of a surface is an intrinsic measure of the curvature of a surface at a point. It is calculated by considering the maximal and minimal curvatures on the surface at a point. Formally, these values are multiplied to give κ . [9]

Theorem (5.5): Given an orthogonal parameterization $f: U \rightarrow S$ of an oriented surface S , where $U \subset \mathbb{R}^2$ is homeomorphic to an open disk and f is compatible with the orientation of S , let $R \subset f(U)$ be a simple region of S , and let $\alpha: I \rightarrow S$ be so that $\partial R = \alpha(I)$. if α is positively oriented, parametrized by arc length

S , and if $\alpha(s_0), \dots, (s_k)$ and $\theta_0, \dots, \theta_k$ are respectively the vertices and external angles of α , then:

$$\sum_{j=0}^k \int_{s_j}^{s_{j+1}} k_g(s) ds + \iint_R k d\sigma + \sum_{j=0}^k \theta_j = 2\pi \quad (4.2)$$

Where k_g is the geodesic curvature of the regular arcs of α and k is the Gaussian curvature of S .

Proof:

Let $u = u(s), v = v(s)$ be the expression of the parameterization of α in the parameterization f .

We have $k_g = \frac{1}{\sqrt{EG}} \left(G_u \frac{dv}{ds} - E_v \frac{du}{ds} \right) + \frac{d\Psi_j}{ds}$

$$\begin{aligned} \sum_{j=0}^k \int_{s_j}^{s_{j+1}} k_g(s) ds &= \sum_{j=0}^k \int_{s_j}^{s_{j+1}} \left(\frac{G_u}{\sqrt{EG}} \frac{dv}{ds} - \frac{E_v}{\sqrt{EG}} \frac{du}{ds} \right) ds + \\ &\sum_{j=0}^k \int_{s_j}^{s_{j+1}} \frac{d\Psi_j}{ds} ds \end{aligned}$$

Now, the Gauss – Green theory states the following: If $P(u, v)$ and $Q(u, v)$ are differentiable functions in a simple region $A \subset \mathbb{R}^2$, whose boundary is given by $u = u(s), v = v(s)$, then

$$\sum_{j=0}^k \int_{s_j}^{s_{j+1}} \left(P \frac{du}{ds} + Q \frac{dv}{ds} \right) ds = \iint_A \left(\frac{dQ}{ds} - \frac{dP}{ds} \right) dudv.$$

Applying this theory, we get:

$$\sum_{j=0}^k \int_{s_j}^{s_{j+1}} k_g(s) ds = \iint_{f^{-1}(R)} \left[\left(\frac{E_v}{\sqrt{EG}} \right)_v + \left(\frac{G_u}{\sqrt{EG}} \right)_u \right] dudv + \sum_{j=0}^k \int_{s_j}^{s_{j+1}} \frac{d\Psi_j}{ds} ds$$

Now since we have an orthogonal parameterization, such that:

$$F = 0,$$

$$\left[- \iint_{f^{-1}(R)} \left[\left(\frac{E_v}{2\sqrt{EG}} + \frac{du}{dt} \right)_v + \left(\frac{G_u}{2\sqrt{EG}} \right)_u \right] dudv \right]$$

$$= - \iint_{f^{-1}(R)} k\sqrt{EG} dudv = - \iint_R kd\sigma$$

And from topology we know the theory of Turning tangents which tells us that:

$$\sum_{j=0}^k \int_{s_j}^{s_{j+1}} \frac{d\Psi_j}{ds} ds = \sum_{j=0}^k k(\Psi_j(s_{j+1}) - \Psi_j(s_j)) = \pm 2\pi \sum_{j=1}^k \Theta_j$$

Since the curve is positively oriented, the sign should be plus. Putting all of this together give us:

$$\sum_{j=0}^k \int_{s_j}^{s_{j+1}} k_g(s) ds + \iint_R kd\sigma + \sum_{j=1}^k \Theta_j = 2\pi [12]$$

Theorem (5.6): Let $R \subset S$ be a regular region of an oriented surface. Let ∂R be made up by closed, piecewise, simple, regular curves: $C_1 \dots C_n$

Then:

$$\sum_{i=0}^k \int_{\sigma_i} k_g(s) ds + \iint_R kd\sigma + \sum_{i=0}^k \Theta_i = 2\pi \chi(R)$$

Proof:

Let J denote a triangulation of R such that each triangle T_j is contained in a neighborhood of orthogonal parametrizations compatible with the orientation of S . We note that such a triangulation exists by the proposition proven above. Now, we simply apply the local Gauss- bonnet theory to each T_j of the above triangulation, and we have:

$$\sum_{i=0}^k \int_{\sigma_i} k_g(s) ds + \iint_R kd\sigma + \sum_{j,k=1}^{F,3} \Theta_{jk} = 2\pi F,$$

Where the indexing of each theta accounts for each external angle of the triangles in J . We note that F here is the number of faces in our triangulation. We denote the interior angles of the triangles by $\Psi_{jk} = \pi - \Theta_{jk}$. We calculate, in general that,

$$\sum_{j,k=1} \Theta_{jk} = 3\pi F - \sum_{j,k=1} \Psi_{jk}$$

We note that the vertices must belong to either some T_j or a G_j , So $V_g = V_{gt} + V_{gc}$, and then, since the sum of the angles around each internal vertex is 2π ,

$$\sum_{j,k=1} \Theta_{jk} = 2\pi E_i + \pi E_g - 2\pi V_i - \pi V_{gt} - \sum_j (\pi - \Theta_i)$$

By adding πE_g and subtracting πE_g to the right hand side of the above equation we have,

$$\sum_{j,k=1} \Theta_{jk} = 2\pi E - 2\pi V - \sum_i \Theta_i$$

Now, we collect the terms to find:

$$\sum_{i=0}^k \int_{c_i} k_g(s) ds + \iint_R k d\sigma + \sum_{i=0}^k \Theta_i = 2\pi(F - E + V).$$

But, by the definition of a triangulation $F - E + V = X(R)$, thus, $= 2\pi X(R)$.

Remark (5.7): We have prove the local case of this theory, and the global theory tells us similar information. We prove this generalization by using the local theory in each triangular region of our triangulation for the given surface. This theory leads to as series very deep corollaries.

Example (5.8): Any smooth surface in R^3 with is homeomorphic to S^2 satisfies: $\int_R k d\sigma = 4\pi$

We already saw this for convex surface using the Gauss map. The surface need not be isometric to S^2 for this relation to hold.[15]

Remark(5.9): We have seen that Gaussian Curvature K of a geometric surface M has a strong influence on other properties of M , notably the shape of M when it is a surface in R^3 . Now we will show that the influence of Gaussian curvature penetrates to the topological conformation of M - to properties independent of the geometry of M . to show this, the main step is a theory that relates the total curvature of a 2- segment to the total amount its boundary curve turns. The geodesic curvature of a curve α in an oriented surface M tells its rate of turning relative to arc length S . So to find the total turning, we integrate with respect to arc length, adjusting suitably when α is merely a regular curve.

Theorem (5.10): If a triangle in a surface is shrinkable to a point, then:

$$2\pi = \int_{\Delta} kg(\Delta) + \int_{\Delta} k + \sum_{j=1}^3 (\pi - \iota_j) \quad (4.6)$$

Moving the angle sum to the other side and multiplying by -1 produces

$$-\int_{\Delta} kg(\Delta) - \int_{\Delta} k = -2\pi + (\pi - \iota_1) + (\pi - \iota_2) + (\pi - \iota_3) = \pi - (\iota_1 + \iota_2 + \iota_3). \quad (4.7)$$

7) [4]

Corollary (5.11): If a triangle Δ in a surface is shrinkable to a point and is made up of geodesic segments, then the sum of the interior angles of the triangle differs from π by (+ or -) the total Gaussian curvature.

6. Some Applications of Gauss – Bonnet: The Gauss – Bonnet theory was proved by cutting an entire surface M into rectangular regions and applying the Gauss – Bonnet formula to each. The scheme works because these rectangles are all consistently oriented by an orientation of M , and thus the integrals $\int kgds$ on their boundaries cancel in pairs. Here in essence is the fundamental idea of algebraic topology- specifically, homology theory. By applying this scheme to suitable regions in M we can extend the range of the Gauss – Bonnet theory.

7.Results:

We found the following results:

The Gauss Bonnet theory relates the Gaussian curvature of a surface to the geodesic curvature , this theory contains many applications within and outside of its original field of differential geometry , these notions of curvature tell us roughly what a surface looks like both locally and globally and the Gauss Bonnet theory is one of the most beautiful and one of the deepest result in the differential geometry of surfaces.

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“Motivations of International Students Who Pursue Post-secondary Education Abroad”

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Final Draft :

“Motivations of International Students Who Pursue Post-secondary Education Abroad”

Target journal: This paper will be formatted in accordance with the requirements for submission to the *International Journal of Lifelong Education*. Considering that this paper will have an international scope in terms of its investigation of what motivates international students to attend universities abroad, this journal is an appropriate choice. The anticipated length of the paper will be about 5,000 words, which is a suitable length for this journal, and the reference style will be APA, as is prescribed by the journal.

Key words :

International students, post-secondary education, motivation, study abroad, transformative learning theory

Introduction

As part of the lifelong learning journey, many students decide to travel to a different country to be educated there and gain exposure to a different culture. This is especially true of students who study English as a second language, as they are often compelled to study internationally, in English-speaking countries, where they can improve their English language skills while also studying a specific subject in English. However, this paper will not be limited to the study of English

exclusively, as it is also the case that English speakers travel abroad to attend university and learn a foreign language, and people with a variety of native languages similarly travel to learn various foreign ones. In some cases, students will even travel to a new country to study in university without having to learn a new language (such as a person from the US studying business while abroad in the UK). Regardless of the language involved, in this project, I will examine the literature on international students and their motivations for studying abroad from the perspective of Jack Mezirow's (1978) transformative learning theory. Basically, I will be arguing that studying abroad and subsequent exposure to a different culture can be transformative for international students, broadening their perspective on the world and altering how they interact with it, a central aspect of transformative learning. The thesis for the project is that attaining a transformative educational experience should be considered a main motivator for international students who decide to study abroad. Over the course of this paper, I will introduce and discuss Mezirow's transformative learning theory with regard to what motivates students to study at universities in foreign countries. Subsequently, I will examine what motivates students from various national contexts—including the United States and Canada for North America, and Japan and China for Asia—to study abroad.

Transformative Learning Theory and Motivation to Study Abroad

As Mezirow (2006) explains in a recent chapter of a book outlining his transformative learning theory, the theory was initially put forth in 1978 in an article titled "Perspective transformation." In that article, Mezirow (1978) basically suggests that learning during adulthood offers adults the opportunity to question their assumptions and re-think their

expectations for life, resulting in a larger shift in terms of what they consider to be meaningful or important to them. Insofar as what we value largely defines us, it was thus argued that adult education or lifelong learning could be a transformative experience, where the adult who enters the lifelong learning program is not the same adult who exits it. Mezirow's (2006) conceptualization of transformative learning theory was largely influenced by the work of Paulo Freire, and especially his concept of conscientization (or a coming to critical consciousness concerning social and political realities), as well as the idea of raising consciousness put forth by the women's rights movement. Overall, the transformation that occurs through education, according to transformative learning theory, is a raising of consciousness. Since Mezirow (2006) initially theorized learning's transformative potential, the theory has become widely popular, resulting in a movement that resulted in various conferences, and articles as well as book publications. However, having examined the literature carefully, it does not appear that any scholars have yet written at length about how international students who decide to study abroad might be motivated by a desire to specifically experience transformative learning. Thus, this research will fill a gap in the existing body of knowledge.

Arguably, this body of knowledge concerning what motivates students to study abroad is particularly important in contemporary times considering the increased globalization that the world is currently experiencing. Globalization has been defined in various ways. However, for the purpose of this paper it is enough to understand that it involves more activities (social, political and economic) occurring across political borders, which results in greater interconnectedness of individuals around the world (Held et al., 1999). The simple fact that we have increasing numbers of international students

is one sign of globalization. One hundred years ago, prior to globalization, the number of international students would be quite small, given the greater difficulty of international travel at that time. However, as the world has grown more interconnected through improved transportation and advanced technologies, students have tended to sometimes travel further away to study. Gaining more insight into the students' motivations for studying abroad could help us to broadly understand the relationship between post-secondary education and globalization. Furthermore, it could help the field of lifelong learning to better motivate adult learners to engage in transformative learning opportunities, whether abroad or locally. This knowledge could also be shared with policy makers at the national or provincial level, assisting them to help motivate more individuals to study abroad.

The links between Mezirow's (2006) transformative learning theory and the motivations of international students who decide to study abroad are suggested by a review of the existing literature. As Mezirow (2006) explains, one of the previous theorists whom he turned to for the foundation of his transformative learning theory was Habermas, who distinguished between two main kinds of learning: instrumental learning and communicative learning. Instrumental learning, which generally involves a hypothesis and deductive logic to assess if the hypothesis is correct or not, helps humans to accomplish a task, such as designing a better boat or discovering the cause of a disease (Mezirow, 2006). Communicative learning, on the other hand, is more arts-related and involves the creation of understandings based on communication, whether that communication be through a conversation or a book (Mezirow, 2006). Whereas instrumental learning aims to discover whether or not something is true, communicative learning aims to judge and

assess in an intellectual and empathetic manner (Mezirow, 2006).

The instrumental and communicative forms of learning outlined by Habermas and influential for Mezirow (2006) can arguably be associated with the motivations that Allen (2010) found to exist among students who studied abroad for the short-term to learn French. Allen (2010) found that these students were motivated to study college-level French in France either for linguistic reasons or to further their career. While career-based motivations are not the same as designing a car or building a house, they indicate a focus on the practical need to get things done, such as getting a job. Thus, the career-based motivations among students can be regarded as being more in line with instrumental learning. On the other hand, the linguistic motivation aligns well with Habermas's communicative learning, where the emphasis is on learning in order to be able to communicate more effectively with and better understand others. While a wide variety of other motivators are discussed throughout this article, we will be consistently returning to this conceptualization, where the desire to be social and communicate is one motivator while more practical concerns are another motivator.

It is also worth noting that Mezirow's transformative learning theory has evolved significantly since it was first described in 1978, and this further development occurred through a series of stages (Kitchenham, 2008). For example, the theory was expanded in 1981 when Mezirow adapted the work of Habermas on three learning domains (technical, practical, and emancipatory) to be applied within the theory, while in 1995 Mezirow decided to place more emphasis on critical self-reflection as a means of transforming one's perspective as part of transformation theory (Kitchenham, 2008). As recent as 2006, Mezirow was continuing to expand

upon his transformative learning theory, making connections between his original theory and constructivist theory as well as the concepts of individuation, psychic distortion and schema therapy (Kitchenham, 2008). Thus, while it is only possible to provide a relatively basic description of the theory in this paper on the motivations of students who decide to study abroad for their post-secondary education, and the aim in this paper is to apply the theory in a relatively simple manner, it should be noted that the theory is in fact quite complex, and further work should be done to relate it to motivators among students studying abroad.

Overview of Motivators to Study Abroad

Mezirow's transformative learning theory is mainly discussed in this paper, allowing for motivators to be roughly divided into those that are career-based and those that support communication with others (in a manner that is not mainly instrumental but focused on developing improved understandings of people in all of their diversity). However, it is notable that other theories can be applied to the topic of students' motivations to study abroad in similar ways. Allen (2010a) employed activity theory in an effort to understand what motivated six American college students studying French (intermediate level) to participate in a six-week study abroad program in France. Data was collected in various ways (such as questionnaires and interviews), and data analysis showed that students were motivated to study abroad for two main reasons (Allen, 2010a). Their motives were either linguistic or career-oriented (Allen, 2010a). These motives can clearly be associated with the communicative and career-oriented motivations described by Mezirow from within the context of transformative learning theory, suggesting that both Mezirow's theory and activity theory arrive at similar conceptualizations of what motivates students to study abroad. This subsequently

suggests that the idea of dividing motivations for studying abroad between those that are communicative or linguistic and those that are more pragmatic and career-oriented is a valid approach. Notably, Allen (2010a) found that those students whose motives were linguistic were more likely to experience enhanced learning of French as a second language as compared to those students who studied French abroad for practical, career-oriented reasons. Thus, this might suggest that learning for practical reasons involves less passionate learning on the part of the student, whereas the desire to communicate creates more of a passion with regards to studying abroad to learn a second language at the college level.

However, it is also important to note that not all motivations to study abroad fits in either the communicative/linguistic or career-oriented categories described by Mezirow (2006) and Allen (2010a). Research conducted by Sanchez et al. (2006) among a sample of American, French and Chinese business students found that a desire to learn additional languages to one's mother tongue was a motivator only among the sample of American students, while Chinese students specifically did not include career-oriented reasons for studying abroad. Instead, among the Chinese sample investigated by Sanchez et al. (2006), either the "search for a new experience" or the "search for travel" were motivational factors encouraging them to study abroad (p. 35). Notably, while this desire to travel and engage in new experiences is clearly not career-oriented but more personally fulfilling in nature, it might be somewhat associated with the communicative motivation, as new experiences and travel—especially in foreign countries—often necessarily involve communication with various individuals in the local context being visited. Nonetheless, the motivation to study abroad as a means of discovering new experiences and travelling might be

considered a motivational factor that is largely independent from the communicative/linguistic and career-oriented motives elaborated upon in this paper.

Another approach that has recently been used in relation to the topic of students' motivation to study abroad has been to view students pursuing undergraduate degrees as emerging adults and consider their motivations in light of this, and specifically in light of Arnett's definition of an emerging adult (Wintre et al., 2015). This study was conducted by Wintre et al. (2015) at a Canadian university among a sample of 64 international students from 26 different countries. Through interviews, data was collected and then underwent thematic analysis to reveal the various motivations that these international students had for studying abroad. The desire for new experiences was one motivational factor found, which is similar to the findings of the research conducted by Sanchez et al. (2006) in terms of the Chinese students' motivations. Education was another factor, which might be related to the communicative/linguistic motivation (Wintre et al., 2015). Meanwhile, career considerations and prospects regarding immigration were also found to be factors, which are clearly related to Mezirow's (2006) and Allen's (2010a) career-oriented motivations. Financial reasons, another motivational factor discovered in this research, might also be considered a pragmatic and career-related motivation (Wintre et al., 2015). However, four other motivations were found that have not been discussed so far in this paper. They include the qualities of the educational institution where students would study, its location (i.e. the foreign country it is situated within), the qualities of the hosting country, and the presence of friends and/or relatives in the host country. Thus, factors associated with the situation abroad where students decide to study also motivate them to study abroad. Some of these factors might be

more related to an interest or personal preference, such as the qualities of the hosting country. While others might be more career-oriented, with some educational institutions offering courses specifically desired to support future career-oriented decisions among students. Furthermore, other factors might be motivators for practical or personal reasons. For example having friends and/or family in a host country could likely make studying there easier in a practical sense (i.e. having someone to show the student studying abroad around their new context, or even having someone to stay with while studying abroad). It could also provide students with an opportunity to simply spend time and become better acquainted with friends and/or relatives in the hosting country.

Other research conducted in the Canadian context involved a sample of 187 university students studying abroad in Canada who were from the regions of Asia, Europe, and Australia/New Zealand (Massey & Burrow, 2012). Data was collected through a survey and found that students were mainly motivated by a desire to live in Canada and develop acquaintances here as well as by a desire to interact with Canadians and learn about the country's culture, all of which can be categorized broadly under Mezirow's communicative motivation category. Notably, this study by Massey and Burrows (2012) found that a desire to be with friends who were also studying abroad as part of the same program was one of the two weakest motivators, suggesting that peer allegiance has little influence, while the other weak motivator was a desire to strengthen ties with family or ethnic heritage. This factor somewhat contradicts Wintre et al.'s (2015) finding that having family in the hosting country was a motivator for students to study abroad. Another factor affecting students' motivation to study abroad was a desire to improve career prospects, a key variable found in Massey and Burrows' (2012)

study. The theorizations of Mezirow (2006) and Allen's (2010a) findings concur with Massey and Burrows (2012).

Motivators to Study Abroad from within the North American Context

As Sanchez et al. (2006) found in their research, motivations for studying abroad can vary between students in different national contexts, therefore this paper focuses on two different regions: the North American and Asian contexts. Originally, I desired to include in this paper a section concerning the motivations of Middle Eastern students who study abroad, but there is a dearth of information on this topic, as no studies seem to have been conducted specifically on Middle Eastern students studying abroad. As Mutua and Sunal (2004) have noted, there is a general lack of research that has been conducted on education in the Middle East as well as other poorer regions of the world, due largely to the definancialization of these regions that has occurred as a consequence of globalization. This may be the issue concerning why no research is available on what motivates Middle Eastern students to study abroad. Although the Middle East might be a poor region in general, Saudi Arabia is a relatively wealthy country that supports many of its students who desire to study abroad, so it is hoped that Saudi researchers might investigate the motivations of Saudi post-secondary students to study abroad in the future. Nonetheless, looking at students from the North American context, Lane-Toomey and Lane (2012) have investigated what motivates American students to study abroad in the Middle East/North African region and Western Europe or Australia, particularly because the number of American undergraduate students deciding to study in this region has increased significantly in recent years. Data was collected from a sample of 275 students who studied abroad in the Middle East/North African region

and 326 in Western Europe or Australia through surveys and focus group discussions (Lane-Toomey & Lane, 2012). Communicative motivation was clearly a major factor among the students who studied abroad in the Middle East/North African region, as 80% of them specified that they were motivated by a desire to develop their foreign language skills (Lane-Toomey & Lane, 2012). However, career-oriented motivations might have been the main factor, as learning to communicate can also benefit one's career. The most important motivation found in this research was a desire to work in the Middle East/North African region for the United States government in some capacity (Lane-Toomey & Lane, 2012). Similarly to Sanchez et al.'s (2006) findings, students were also motivated by a desire to have different experiences, develop an understanding of a novel culture, or simply to have fun (Lane-Toomey & Lane, 2012). The desire to have a unique cultural experience was specified by the students as the reason they chose to study abroad in the Middle East/North African region rather than in, for example, Europe or Australia (Lane-Toomey & Lane, 2012).

A study conducted by Li et al. (2013) used a slightly different approach. They investigated the motivations of 555 study participants who were university students pursuing undergraduate degrees and who did or did not intend to study abroad (but who were not actually studying abroad at the time the study was conducted)(Li et al., 2013). The study found, by analyzing data collected through a survey, that students who wished to achieve a great deal academically (perhaps as an aspect of desiring to secure a good career in the future) were more motivated to study abroad (Li et al., 2013). Notably, however, this achievement motivation was found in post-hoc analysis to only significantly affect the desire of males to study abroad, but not females (Li et al., 2013).The students who

wanted to study abroad also showed high scores for neophilia, or a preference for change and unusual things or experiences, which could be related to wanting to study abroad in order to have unique experiences (Li et al., 2013). In addition, the study found that those who wished to study abroad had a greater desire to help others, suggesting that this could also be a key motivator, which would be placed in the communicative motivation category as helping is usually an inherently communicative action (Li et al., 2013). Even among students who have not yet studied abroad, it is suggested that they are motivated to study abroad in the future by similar reasons as those who have actually moved forward with plans to study abroad (Li et al., 2013).

While Li et al. (2013) provide useful insights concerning what motivates students to study abroad, it is also worth considering what characteristics were associated with students who did not desire to study abroad. For individuals or organizations promoting study abroad programs, these characteristics associated with those who do not desire to study abroad could be regarded as challenges to be overcome or, at least, demographic factors to be aware of. The participants in Li et al.'s (2013) research with a low desire to study abroad had relatively low achievement motivation, a low level of neophilia, less desire to help others, and lacked the migrant personality. Thus, individuals with these characteristics would be less likely to want to participate in a study abroad program.

While most of the research found on this topic for North American students was focused on students from the United States, Knight and Madden (2010) used a sample of Canadian doctoral students in the Social Sciences and Humanities for their research. Among this sample of students, data was collected on their reasons for being interested in academic mobility, which is very similar to motivations for studying in a

different national context or abroad (Knight & Madden, 2010). Some of the reasons were directly career-oriented, such as enhancing one's career path and seeking employment, while others were academic-oriented. This could also relate to one's future career, as was the case with attending conferences/workshops, engaging in collaborative research, taking special courses, gaining access to a foreign supervisor, collecting data or performing field work, and gaining access to special equipment (Knight & Madden, 2010). However, learning a language was also cited as a reason for being interested in academic mobility, suggesting a communicative/linguistic motivation, while a desire to seek out new experiences in life was also mentioned as one of the top ten reasons (Knight & Madden, 2010).

Motivators to Study Abroad from within the Asian Context

As a number of articles have suggested, the motivations of Asian students to study abroad for all or part of their post-secondary education can be quite different from those expressed by North American students. Yan and Berliner (2011) note that a general attitude in China that the West is more advanced pulls students towards studying in the West, especially in an effort to gain prestige or other instrumental reasons. Low incomes, sub-par living conditions and a dearth of professional openings, as well as instability in the political sphere, have also been suggested as reasons that motivate Chinese students to study in the United States specifically (Yan & Berliner, 2011). As Wu (2014) found in a study involving a sample of 169 students from mainland China studying in the United Kingdom, conditions in China (seen negatively) as compared to conditions in other countries, such as the United Kingdom (seen positively) were indeed a motivating factor, in agreement with the report by Yan and

Berliner (2011). However, in addition Wu (2014) found that older Chinese students and those in Master of Arts programs in the United Kingdom were more likely to be motivated by a desire to experience novel cultures. Meanwhile, younger students and those pursuing Master of Science degrees were more likely to be motivated by academic concerns (Wu, 2014).

Further research conducted by Kormos et al. (2014) among international students studying abroad in the United Kingdom involved mostly Chinese students (73%) out of a sample of 70 international students. For the interview part of this study, six out of ten were Chinese (Kormos et al., 2014). In this study, it was found that students were very focused on language-learning as the main motivational factor, specifically because English is regarded as a “global language” that individuals should make an effort to learn (Kormos et al., 2014, p. 157). While this would most obviously be classified as a communicative motivation, it is noteworthy that Kormos et al. (2014) position the language-learning motivation as instrumental. This is likely due to the fact that learning the English language specifically, due to its global prominence, is more likely to expand various career opportunities, making learning English a particularly pragmatic motivation.

It is worth noting that while the number of students choosing to study abroad has been generally increasing in recent years, one Asian country, Japan, has remarkably seen a decline in the number of students studying abroad (Lassegard, 2013). As part of an investigation into why this might be the situation, Lassegard (2013) collected survey data from a sample of 328 Japanese university students, asking them about their reasons for wishing to study abroad. Similarly to the Chinese students discussed above, development of foreign language skills was a main motivating factor for 62% of the students. However, an even greater number of students

(67.6%) responded that a desire to become more global minded was their motivation, while 38.5% of students were motivated by a desire to learn about a different culture (Lassegard, 2013).

The Advantages of Certain Motivators for Studying Abroad

While this paper thus far has discussed the various motivations that students from various regions might have for studying abroad during their post-secondary education, it is important to also consider how certain motivations might be more advantageous than others. Chirkov et al. (2007) conducted two studies among Chinese international students studying in Canada and Belgium, using the Self-Regulation Questionnaire, and arrived at some interesting findings. These researchers hypothesized that self-determined motivation to study abroad among the students would benefit them more than non-self-determined motivation, which was supported by the study's results (Chirkov et al., 2007). The study found that when students studied abroad in order to escape negative conditions in their home country, considered a "preservation factor," these students did not adapt well to the culture of their host country, as they arguably felt relatively forced to study abroad and escape negative conditions (Chirkov et al., 2007). However, when the "self-development factor" played a dominant role, so students were motivated to study abroad in order to gain a good education or develop better career opportunities, then these self-determined motivations resulted in students adapting better to their hosting countries (Chirkov et al., 2007). Thus, when students self-determine their motivations for studying abroad, this, as these studies suggest, tends to help them adapt to the culture of their hosting country (which could have a positive impact on their educational experience in that country).

Conclusion

Over the course of this paper, a number of motivating factors have been discussed in relation to post-secondary students engaging in studies abroad. As per the theory of Mezirow (2006) and Allen's (2010a) findings, motivations are generally divided into communicative/linguistic and career-oriented categories. The research reviewed in this paper generally supports the idea that these are two main categories for motivations. However, the research also suggests that other motivational factors can have an influence. Some students might be motivated by other positive factors, such as a desire to experience unique cultures, spend time with family and/or friends, travel, or simply have fun. On the other hand, some students might be compelled to study abroad due to disadvantageous conditions in their home country, such as poor living conditions and a lack of employment opportunities, which explains some of the differences between students from different national contexts in terms of their motivations for studying abroad. Future research should continue to investigate the impact that various motivations have on the success of students studying abroad. Furthermore, there are regions and countries where there is a lack of research concerning what motivates students to study abroad, such as the Middle Eastern region. No research was found during the research for this paper concerning what motivates Middle Eastern students to study abroad for post-secondary education, so research should be conducted in this area to fill this gap in our knowledge.

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Gold Mineralization of The Area around Hamid and Elkhanag Villages (Sukkot district) Northern Sudan **Younis Suliman Mustafa³**

Abstract

Gold deposit in the area occur in quartz veins and as fine gold in sandstone. Gold bearing quartz veins of the target area occur mainly in east-west trending faults system and fractures cut in green stone belts. Gold mineralization is mainly confined to the metovolcanic, metasedimentary rocks of green schist facies (schist's) intruded by granitoid rocks, and overlain by Nubian sandstone which exposed at the western side of the area. Nubian sandstone contain fine gold accumulated by the River Nile through slow currents forming ancient beach and terraces. The quartz veins contain metal sulfide deposits such as pyrite. The ore have been largely mined by local Artisanal miners. Chip samples of quartz veins show grade between 1.4 - 58.4 ppm, and the sandstone between 1 - 25 ppm. Visible gold also observed as nuggets, plates, and fine grains.

Introduction

Location and accessibility

The prospected area is located in Northern Sudan West of the Nile between Hamid and Elkhanag Villages. The area bounded by latitude ($20^{\circ} 36' 12'' - 20^{\circ} 39' 44''$)N and longitude ($30^{\circ} 14' 03'' - 30^{\circ} 18' 30''$) E. fig. (1).

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Fig (1): location map of the study area.

The target area is accessible from Khartoum by asphaltic highway road via Dongola to Argean. It is about 175 Kilometer North Dongola town.

Regional geologic setting

The area of interest is a part of the Northern Sudan, related to late Proterozoic Arabian – Nubian shield at the western bank of the Nile at the border with the old African craton. The general agreement is that the ANS represents the best documented example of the late Proterozoic to early Proterozoic crustal growth through process of the lateral change of area and terrain accretion (1). The gold deposits in Sudan are all contained in quartz veins which are wide spread throughout the basement area. The basement sequences are sedimentary and volcanic origin and are regionally metamorphosed in green schist to amphibolite facies.

There is local evidence of late or post tectonic granitic plutons emplaced in the surrounding of auriferous deposits. The gold -bearing quartz veins are considered to be associated with a regional fraction which postdate the orogenesis and metamorphism of the basement.

The rocks are affected by low grade metamorphism of green schist facies.

The geology of gold mineralization of the area

Primary gold:

The area is underlain by a variety of Precambrian basement complex rocks that include metavolcanic and metasedimentary sequences. The quartz veins almost occur beneath the surface with depth extended up to 30 meters in three bore holes and almost covered by quaternary deposits (mainly clays and kaolin) up to 4 meters thick, and the quartz veins were discovered by digging pits.

The quartz veins were hosted in a sequence of chlorite schist, carbonate - mica schist, quartz- mica schist, which is of lower green schist facies. The country rocks were strongly folded and sheared. The dip of fold axes take different directions and the contact zone between the quartz veins and adjacent host rocks show ferruginous alteration.

The gold-bearing quartz veins are mainly lenses-like, very often showing bodinages giving vertical and lateral change in geometry and composition. The veins generally occur subparallel to the lithological banding being folded together with the host rocks, which have general North and East strike. The gold-bearing quartz veins commonly occupy specific structural features as minor shear zones, fold hinges, and/or fracture network (2). All the quartz are in exo-contact. The regional trend of volcano-sedimentary sequences are occupied by either highly folded, strong sheared, and brecciated quartz veins.

The gold bearing quartz veins are bluish- gray white, white brown, gray white in colors.

Gold mineralization is hosted by quartz veins which contain minor sulphides such as pyrite, and chalcopyrite, the fine grained gold is also associated with hematite and iron hydroxides resulting from sulfide oxidization and leaching.

The auriferous are sheets consists of veins, lenses, pods and quartz stringers. Gold in the oxidation zone are partly included in quartz hematite ore, Fe-hydroxides, partly as free grains in pores and fissure.

Most of the quartz veins occur in carbonate rich pyritiferous mica schist, chlorites schist, with small bands of mylonite.

The veins are general in order of ten of decimeters to 2 meter wide and spacing from half to one meters. The quartz veins are variable in length, the maximum length discontinuous about 100 meters.

Placer gold and Nubian sand stone gold mineralization

Anomalously high gold contents were found in wadies near the old mines workings along the main quartz veins. Gold also occur in Wadi Sediments of the study areas in as expression of Quaternary paleo-climate, have alternating periods of relatively humind, semi-arid and fully arid conditions. Placer gold have been mined largely by the Artisanal miners through screening and panning. In some samples remarkable content of visible gold have been observed.

Gold also have been recorded in well sorted Nubian sand stone (this is a new result) in pit digged by the Artisanal in the NSS up to 40 meters deep. This NSS overlain the basement rocks forming old beach of the River Nile, extended parallel to the Nile for about 30kms long This sandbeach is deposited by water by the action of slow currents at arid climate condition. The sandstone shows crossbedding, and ripple marks. It's fine grains cemented by calcium carbonate, and iron oxides. The pit was sampled through depth every 10 meter intervals up 40 meter deep. It's found that gold content increase by depths, the result as follows 1.414, 2.12, 5, and 25 ppm respectively. This thick layer of sandstone indicate that gold concentration expended laterally along the bank of the River Nile. The gold concentration is not occur along an old channel, or accumulated by rapped flods for the lack of graded bedding or any fanglomeratic sediments observed.. Gold in NSS could be as a result of more than one cycle of dispersion and then sorting. A combination of these processes their interrelation at or near the surface, since mineralization times in Precambrian, govern gold dispersion and

migration and concentration. The River Nile itself since its first appearance on this part of the crust has changed its course and direction several times from west to east forming old beaches and terraces. The present dispersion of gold and chemical element in the area is not due to present day climatic conditions but due to paleogechemical cycles of trace elements. This result encouraging for further prospection and exploration work on the NSS gold.

Sampling and results

The samples have been selected from the ore and collected from pits and shafts of Artisanal miners in the mineralized quartz veins. The location of samples of shown by the coordinates, with help of GPS. Covering an area of approximately 40 Km².

The chip samples and pit samples from Nubian sandstone was analysed in Ministry of Minerals, giving anomalous values, table (1). Gold values ranges between 1.4 pph and 58.4 ppm. The high values are definitely encouraging for further systematic prospecting and exploration work for gold and associated minerals such as Cu, Pb and Zn.

Visible gold also observed as nuggets, plates, and fine grains in hard specimens, also as free, fine grains, then stringer or intergranular platelets, and as inclusions in iron oxides.

Table (1): AAS analysis of gold bearing quartz veins samples

Sample Number	Location	Latitude	Longitude	PPM
1	Elkhanag	20° 35' 32" N	30° 16' 50" E	29.29
2	Elkhanag	20° 36' 30" N	30° 16' 44" E	1.4425
3	Hamid	20° 39' 02" N	30° 17' 11" E	14.56
4	Hamid	20° 39' 13" N	30° 17' 22" E	58.41
5.1	Hamid	20° 39' 44" N	30° 14' 03" E	6.692
5.2	West Hamid	20° 39' 44" N	30° 14' 03" E	2.12

Table (2): result of the pit samples of gold from top to bottom

Sample No	Depth m	Results ppm
1	0	1.414
2	10	2.12
3	20	5
4	30	25
5	40	25

Conclusion

The area is a part of the Arabian- Nubian shield at the contact with the old African craton. The area related to the late Proterozoic, it is of volcano-sedimentary rocks affected by low-grade metamorphism of green schist facies.

Gold mineralization is hosted by quartz veins which contain minor sulphides such as pyrite, and chalcopyrite together with free (native)gold grains in fissures and pores, also gold is recorded in sandstone as fine grains accumulated by the River Nile.

Chemical analysis of chip samples from quartz veins show high concentration. Nubian sandstone must be subjected to systematic sampling in grid way laterally and through depth all along the beach.

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