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The Techniques for Solving Nonlinear Fractional Reduction-Diffusion Equations

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Abstract: In this paper, Adomian's Decomposition Method (ADM), was applied to solving nonlinear fractional redaction-diffusion equations. Caputo's describes the fractional derivatives. Study four fractional partial differential equations, heat equation, Burger's equation, Fisher's equation and Burgers-Fisher equation. By this method, the solution considered as the sum of an infinite series. In addition, our graphical represented of the solutions have been given by using MATLAB program.

Keywords: Fractional calculus; Adomian's decomposition method; Diffusion equations.

Introduction:

Will study the nonlinear revolution equations where each equation contains the dissipative term u_{xx} in addition to other partial derivatives. This new family of nonlinear equations obtained from scientific applications and physical phenomena. In this paper, use Adomian decomposition method for solving nonlinear reduction–diffusion equations. The new family of nonlinear equations that will be discuss in this paper, defined as

$$D_t^{\alpha} u - v u_{xx} = p(u), \tag{1}$$

where $u(x,t)$ is a function of space x and time t, v is constant viscosity and $p(u)$ is nonlinear term (Lokenath Debnath, 2007). If:

i. $p(u) = 0$, we obtain the linear diffusion equation.
ii. $p(u) = -uu_x$, we obtain the Burgers equation.
iii. $p(u) = ku(1-\frac{u}{k})$, we obtain the Fisher's equation.
iv. $p(u) = uu_x + ku(1-\frac{u}{k})$, we obtain the Burgers-Fisher equation.

1 Preliminaries

In this section, gives some important definitions

Definition (1): Gamma Function

$$\Gamma(n)=\int_{0}^{\infty} t^{n-1}e^{-t}dt=(n-1)!\,,\;\; n\in IN \tag{2}$$

Which is convergent for $n>0$, (Murray R. Spiegel, 1973). A recurrence formula for gamma function are

$$\Gamma(n+1)=n\Gamma(n) \quad for \;\; n\in IR^{+} \tag{3}$$

$$\Gamma(n)=\frac{\Gamma(n+1)}{\mathrm{n}} \quad for \;\; n\in IR^{-} \tag{4}$$

Definition (2): Riemann-Liouville Fractional Integral Operator

Suppose that $\alpha>0$, $n-1<\alpha\le n$, the Riemann-Lioville fractional integral define as (Joseph M. Kimeu, 2009)

$$D^{-\alpha}\big(f(t)\big)=\frac{1}{\Gamma(\alpha)}\int_{a}^{t}(t-u)^{\alpha-1}f(u)du \tag{5}$$

Fractional integral for polynomial and fractional derivative

$$D^{-\alpha}(t^{n})=\frac{\Gamma(n+1)}{\Gamma(\alpha+n+1)}t^{\alpha+n} \tag{6}$$

$$D^{-\alpha}\big(D^{\alpha}f(t)\big)=f(t)-\sum_{k=0}^{n-1}\frac{t^{k}}{k!}f^{(k)}(0), \quad n-1<\alpha\le n \tag{7}$$

Definition (3): Caputo Fractional Differential Operator

Suppose that $\alpha>0$, $n-1<\alpha\le n$, the Caputo fractional differential define as (Joseph M. Kimeu, 2009)

$$D_{c}^{-\alpha}\big(f(t)\big)=\frac{1}{\Gamma(n-\alpha)}\int_{a}^{t}\frac{f^{n}(u)}{(t-u)^{\alpha-n+1}}du \tag{8}$$

2 Adomian's Decomposition method

The Adomian's Decomposition method (ADM) gives rapidly convergent successive approximations of the exact solution if such a solution exists. In what follows, summarize the main steps of this method. For the differential equation

$$Lu+Nu=g(x,t) \tag{9}$$

where L and N are linear and nonlinear operators respectively, and $g(x,t)$ is the source inhomogeneous term, and F is nonlinear function (Khaled Abdalla, 2019)

$$Lu=g(x,t)-Nu \tag{10}$$

$$L^{-1}Lu = L^{-1}(g(x,t) - Nu) \tag{11}$$

$$u(x,t) = \phi_0 + L^{-1}(g(x,t) - Nu) \tag{12}$$

$$\phi_0 = \begin{cases} u(x,0) & for\ L = \dfrac{\partial}{\partial t} \\ u(x,0) + tu_t(x,0) & for\ L = \dfrac{\partial^2}{\partial t^2} \\ u(x,0) + tu_t(x,0) + \dfrac{t^2}{2}u_{tt}(x,0) & for\ L = \dfrac{\partial^3}{\partial t^3} \end{cases} \tag{13}$$

The solution is given by

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t) = \phi_0 + L^{-1}\big(g(x,t)\big) - L^{-1}\sum_{n=0}^{\infty} A_n \tag{14}$$

where the so-called Adomian polynomials A_n. The Adomian polynomials A_n for the nonlinear term *F*(*u*) can be evaluated by using the following expression

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[F\left(\sum_{j=0}^{n} \lambda^j\, u_j\right)\right]_{\lambda=0} , n = 0,1,2,\ldots \tag{15}$$

We obtain

$$A_0 = F(u_0) \tag{16}$$

$$A_1 = u_1F'(u_0) \tag{17}$$

$$A_2 = u_2F'(u_0) + \frac{1}{2}{u_1}^2F''(u_0) \tag{18}$$

$$A_3 = u_3F'(u_0) + u_1u_2F''(u_0) + \frac{1}{6}{u_1}^3F'''(u_0) \tag{19}$$

$$A_4 = u_4F'(u_0) + \left(\frac{1}{2}{u_2}^2 + u_1u_3\right)F''(u_0) + \frac{1}{2}{u_1}^2u_2F'''(u_0) + \frac{1}{24}{u_4}^3F''''(u_0) \tag{20}$$

3 Numerical Results

In this section, stratify the Adomain decomposition Method (ADM) for Heat equation, Burgers equation, Fisher equation and Burgers-Fisher equation.

Example 1. Heat equation

$$D_t^{\alpha}u = u_{xx}\ , \quad t > 0,\ \ 0 < \alpha \leq 1 \tag{21}$$

$$\text{IC:} \quad u(x,0) = x^2 \quad , \ \ 0 < x < 10$$

Integration both sides, obtain

$$D_t^{-\alpha} D_t^{\alpha} u = D_t^{-\alpha} u_{xx}$$

$$u(x,t) - \sum_{k=0}^{n-1} \frac{t^k}{k!} \frac{\partial^k u(x,0)}{\partial t^k} = D_t^{-\alpha}(u_{xx})$$

First Non-fractional, consider $\alpha = 1$,

$$u(x,t) - u(x,0) = D_t^{-1}(u_{xx})$$

Define the ADM

$$\sum_{m=0}^{\infty} u_m = u(x,0) + D_t^{-1}(\sum_{m=0}^{\infty} u_m)_{xx}$$

Suppose

$$u_0 = u(x,0) = x^2$$

$$u_1 = D_t^{-1}(u_0)_{xx} = 2t$$

$$u_2 = D_t^{-1}(u_1)_{xx} = 0$$

$$u_3 = D_t^{-1}(u_2)_{xx} = 0$$

$$u(x,t) = u_0 + u_1 + u_2 + \cdots = x^2 + 2t$$

Second with-fractional, consider $\alpha = 0.5$,

$$u(x,t) - u(x,0) = D_t^{-0.5}(u_{xx})$$

Define the ADM

$$\sum_{m=0}^{\infty} u_m = u(x,0) + D_t^{-0.5}(\sum_{m=0}^{\infty} u_m)_{xx}$$

Suppose

$$u_0 = u(x,0) = x^2$$

$$u_1 = D_t^{-0.5}(u_0)_{xx} = 4\sqrt{\frac{t}{\pi}}$$

$$u_2 = D_t^{-1}(u_1)_{xx} = 0$$

$$u_3 = D_t^{-1}(u_2)_{xx} = 0$$

$$u(x,t) = u_0 + u_1 + u_2 + \cdots = x^2 + 4\sqrt{\frac{t}{\pi}}$$

Table 1: The result when time is 1.5 and the length is 10 cm

x	$\alpha = 1$	$\alpha = 0.5$
0.0000	**0.00200**	**0.07140**
1.0000	**1.00200**	**1.07140**
2.0000	**4.00200**	**4.07140**
3.0000	**9.00200**	**9.07140**
4.0000	**16.0020**	**16.0714**
5.0000	**25.0020**	**25.0714**
6.0000	**36.0020**	**36.0714**
7.0000	**49.0020**	**49.0714**
8.0000	**64.0020**	**64.0714**
9.0000	**81.0020**	**81.0714**
10.0000	**100.0020**	**100.0714**

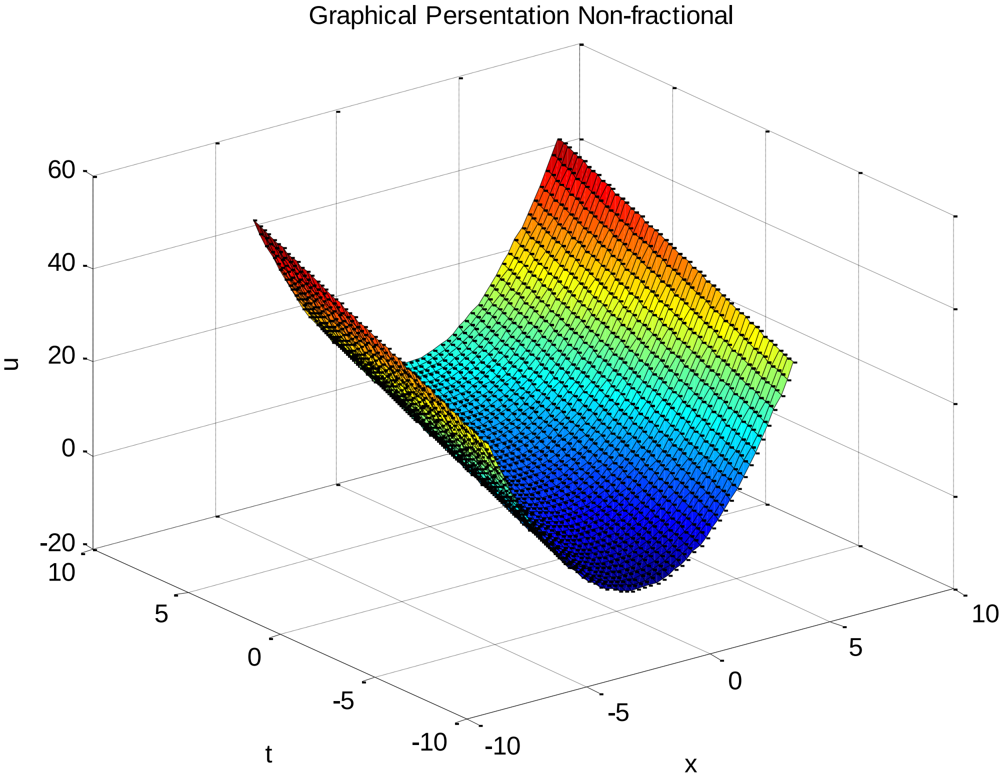

Figure 1: Graphical Presentation Non Fractional of Heat Equation

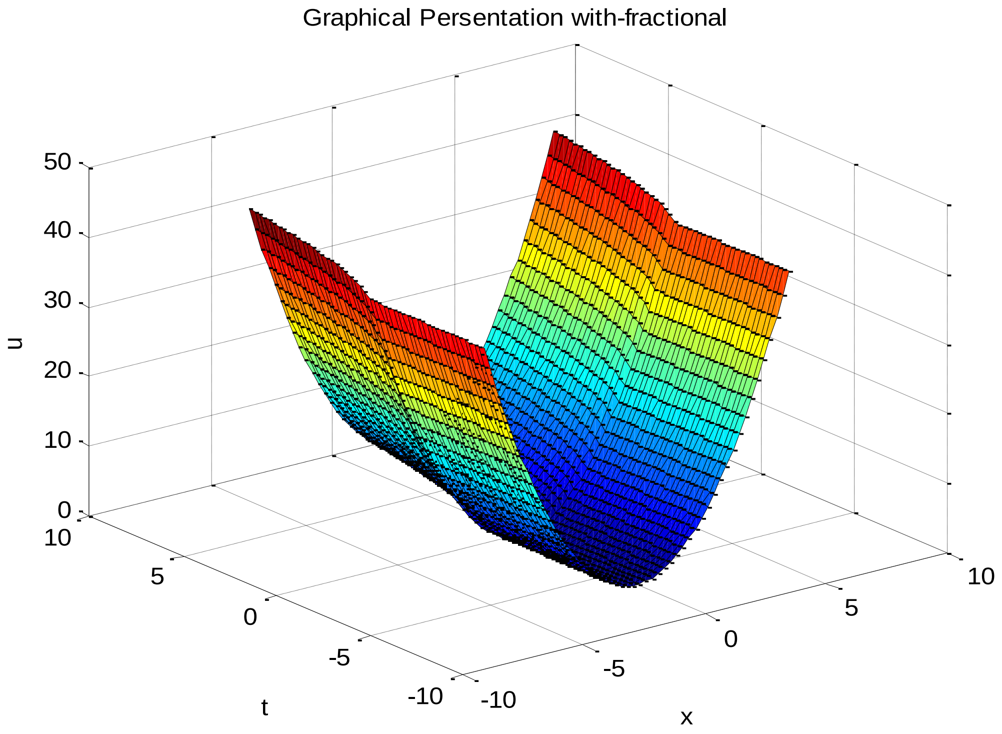

Figure 2: Graphical Presentation with Fractional of Heat Equation

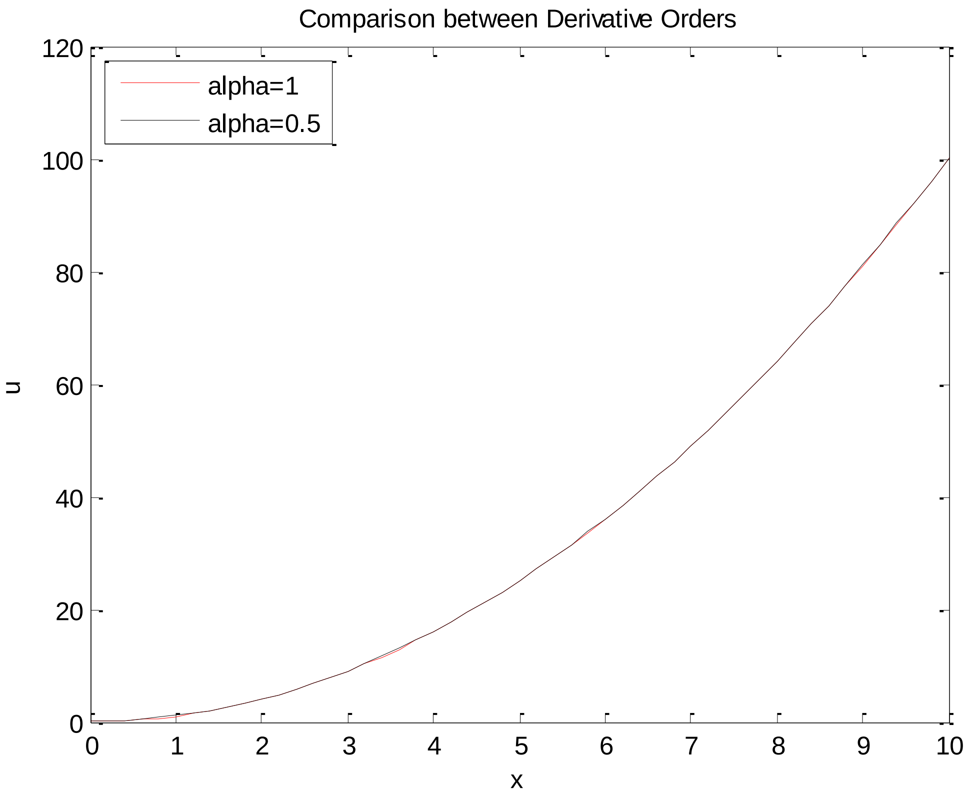

Figure 3: Comparison between Derivative Orders of Heat Equation

Example 2. Burgers equation

$$D_t^{\alpha} u + u u_x - v u_{xx} = 0 \quad , v > 0, , t > 0, \; 0 < \alpha \leq 1 \tag{22}$$

$$\text{IC:} \quad u(x,0) = x \quad , \; 0 < x < L$$

Integration both sides, obtain

$$D_t^{-\alpha} D_t^{\alpha} u = D_t^{-\alpha}(vu_{xx} - uu_x)$$

$$u(x,t) - \sum_{k=0}^{n-1} \frac{t^k}{k!} \frac{\partial^k u(x,0)}{\partial t^k} = D_t^{-\alpha}(vu_{xx} - uu_x)$$

First Non-fractional, consider $\alpha = 1$ and $v = 1$,

$$u(x,t) - u(x,0) = D_t^{-1}(u_{xx} - uu_x)$$

Define the ADM

$$\sum_{m=0}^{\infty} u_m = u(x,0) + D_t^{-1}[(\sum_{m=0}^{\infty} u_m)_{xx} - \sum_{m=0}^{\infty} A_m]$$

$$A_m = \frac{1}{m!} \frac{d^m}{d\lambda^m} \left[\left(\sum_{j=0}^{m} \lambda^j u_j \right) \frac{\partial}{\partial x} \left(\sum_{j=0}^{m} \lambda^j u_j \right) \right]_{\lambda=0}$$

Suppose

$$u_0 = u(x,0) = x$$

$$u_1 = D_t^{-1}[(u_0)_{xx} - A_0] = D_t^{-1}[(u_0)_{xx} - u_0(u_0)_x] = D_t^{-1}[0 - x] = -xt$$

$$u_2 = D_t^{-1}[(u_1)_{xx} - A_1] = D_t^{-1}[(u_1)_{xx} - u_0(u_1)_x - u_1(u_0)_x]$$

$$u_2 = D_t^{-1}[0 + xt + xt] = xt^2$$

$$u_3 = D_t^{-1}[(u_2)_{xx} - A_2] = D_t^{-1}[(u_2)_{xx} - u_0(u_2)_x - u_1(u_1)_x - u_2(u_0)_x]$$

$$u_3 = D_t^{-1}[0 - xt^2 + xt^2 - xt^2] = -\frac{1}{3}xt^3$$

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$$u(x,t) = u_0 + u_1 + u_2 + u_3 + \cdots \cong x - xt + xt^2 - \frac{1}{3}xt^3$$

Second with-fractional, consider $\alpha = 0.5$ and $v = 1$,

$$u(x,t) - u(x,0) = D_t^{-0.5}(vu_{xx} - uu_x)$$

Define the ADM

$$\sum_{m=0}^{\infty} u_m = u(x,0) + D_t^{-0.5}[(\sum_{m=0}^{\infty} u_m)_{xx} - \sum_{m=0}^{\infty} A_m]$$

$$A_m = \frac{1}{m!}\frac{d^m}{d\lambda^m}\left[\left(\sum_{j=0}^{m}\lambda^j u_j\right)\frac{\partial}{\partial x}\left(\sum_{j=0}^{m}\lambda^j u_j\right)\right]_{\lambda=0}$$

Suppose

$$u_0 = u(x,0) = x$$

$$u_1 = D_t^{-0.5}[(u_0)_{xx} - A_0] = D_t^{-0.5}[(u_0)_{xx} - u_0(u_0)_x] = D_t^{-0.5}[0 - x] = -2x\sqrt{\frac{t}{\pi}}$$

$$u_2 = D_t^{-0.5}[(u_1)_{xx} - A_1] = D_t^{-0.5}[(u_1)_{xx} - u_0(u_1)_x - u_1(u_0)_x]$$

$$u_2 = D_t^{-0.5}\left[0 + 4x\sqrt{\frac{t}{\pi}}\right] = 2xt$$

$$u_3 = D_t^{-0.5}[(u_2)_{xx} - A_2] = D_t^{-0.5}[(u_2)_{xx} - u_0(u_2)_x - u_1(u_1)_x - u_2(u_0)_x]$$

$$u_3 = D_t^{-0.5}\left[0 - 4x(1+\frac{1}{\pi})t\right] = -\frac{16}{3\sqrt{\pi}}4x(1+\frac{1}{\pi})t^{\frac{3}{2}}$$

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$$u(x,t) = u_0 + u_1 + u_2 + u_3 + \cdots \cong x - 2x\sqrt{\frac{t}{\pi}} + 2xt - \frac{16}{3}4x(1+\frac{1}{\pi})\sqrt{\frac{t^3}{\pi}}$$

Table 2: The result when time is 0.001, the length is 10 cm and $v = 1$

x	$\alpha = 1$	$\alpha = 0.5$
0.0000	**0.00000**	**0.00000**
1.0000	**0.9990**	**0.9662**
2.0000	**1.9980**	**1.9324**
3.0000	**2.9970**	**2.8986**
4.0000	**3.9960**	**3.8648**
5.0000	**4.9950**	**4.8310**
6.0000	**5.9940**	**5.7972**
7.0000	**6.9930**	**6.7633**
8.0000	**7.9920**	**7.7295**
9.0000	**8.9910**	**8.6957**
10.0000	**9.9900**	**9.6619**

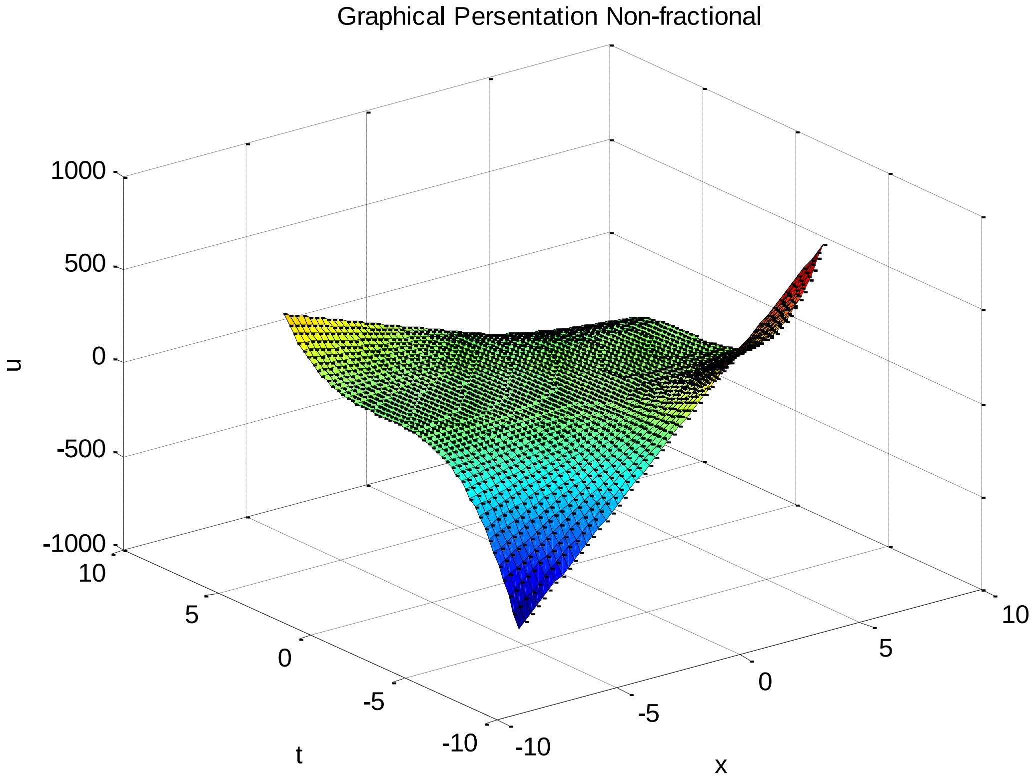

Figure 4: Graphical Presentation Non-Fractional of Burger's Equation

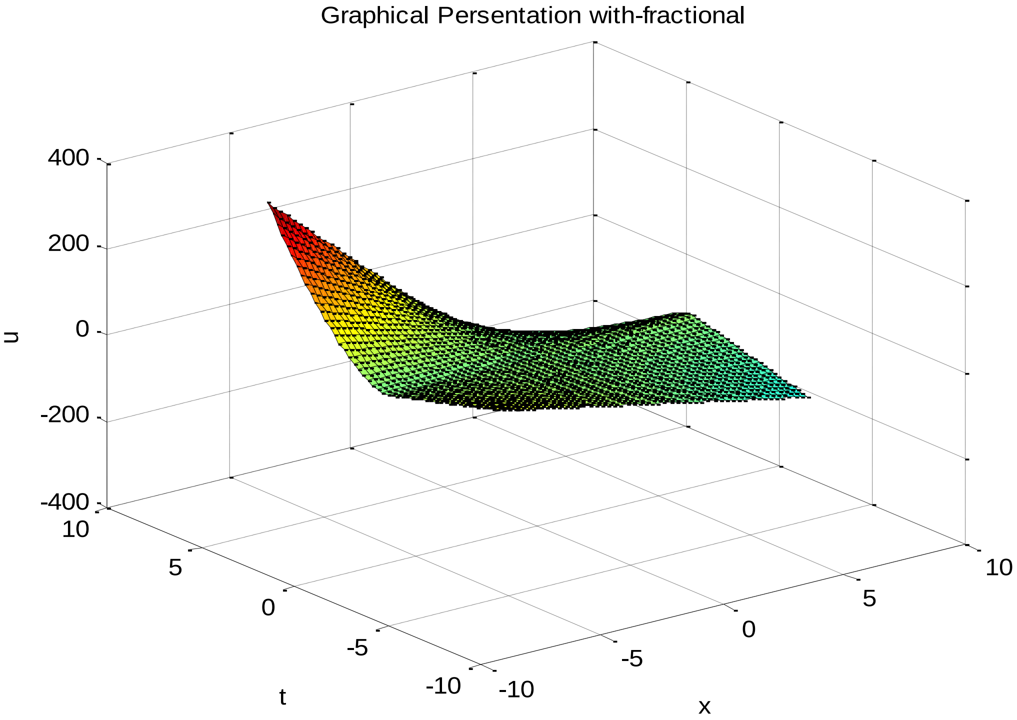

Figure 5: Graphical Presentation with Fractional of Burger's Equation

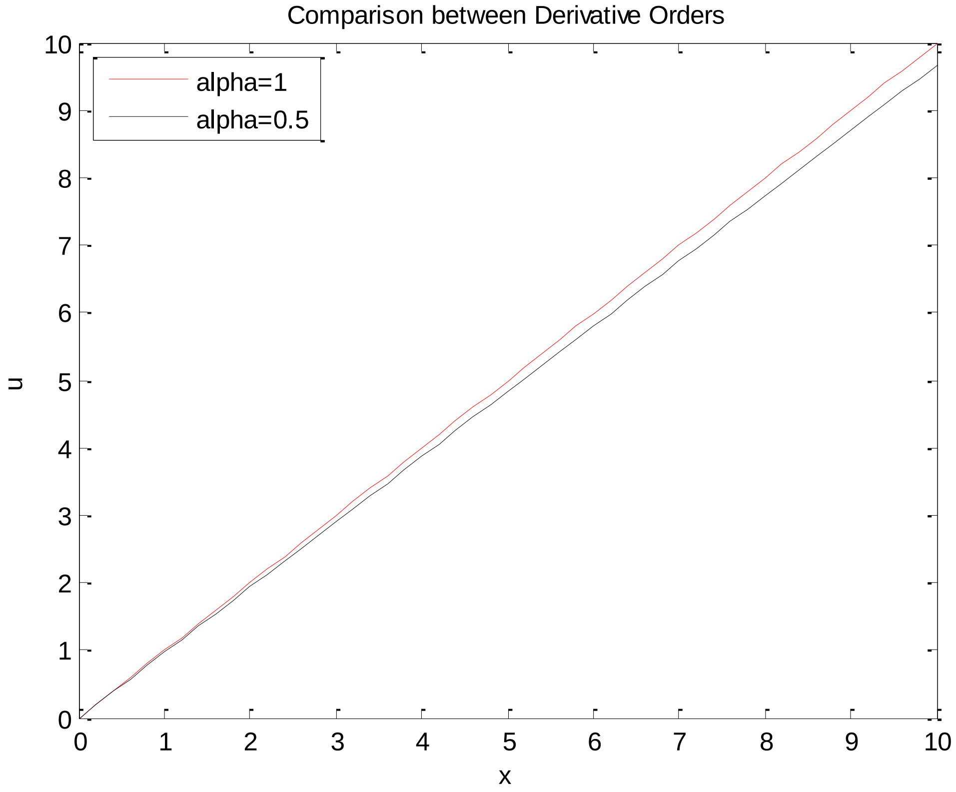

Figure 6: Comparison between Derivative Orders of Burger's Equation

Example 3. Fisher's equation

$$D_t^{\alpha} u = v u_{xx} + k u \left(1 - \frac{u}{k}\right), \qquad k > 0 \ \& \ v > 0, t > 0, \ \ 0 < \alpha \leq 1 \qquad (23)$$

$$\text{IC:}\quad u(x,0) = x \quad , \quad 0 < x < L$$

Integration both sides, obtain

$$D_t^{-\alpha} D_t^{\alpha} u = D_t^{-\alpha}(v u_{xx} + ku\left(1 - \frac{u}{k}\right))$$

$$u(x,t) - \sum_{k=0}^{n-1} \frac{t^k}{k!} \frac{\partial^k u(x,0)}{\partial t^k} = D_t^{-\alpha}(v u_{xx} + ku\left(1 - \frac{u}{k}\right))$$

First Non-fractional, consider $\alpha = 1$, $k = 1$ and $v = 1$,

$$u(x,t) - u(x,0) = D_t^{-1}(u_{xx} + u - u^2)$$

Define the ADM

$$\sum_{m=0}^{\infty} u_m = u(x,0) + D_t^{-1}[(\sum_{m=0}^{\infty} u_m)_{xx} + \sum_{m=0}^{\infty} u_m - \sum_{m=0}^{\infty} A_m]$$

$$A_m = \frac{1}{m!} \frac{d^m}{d\lambda^m} \left[\left(\sum_{j=0}^{m} \lambda^j \, u_j \right)^2 \right]_{\lambda=0}$$

Suppose

$$u_0 = u(x,0) = x$$

$$u_1 = D_t^{-1}[(u_0)_{xx} - A_0] = D_t^{-1}[(u_0)_{xx} + u_0 - u_0^2] = D_t^{-1}[0 + x - x^2] = (x - x^2)t$$

$$u_2 = D_t^{-1}[(u_1)_{xx} + u_1 - A_1] = D_t^{-1}[(u_1)_{xx} + u_1 - 2u_0 u_1]$$

$$u_2 = D_t^{-1}[-2t + (x - x^2)t + 2x(x - x^2)t]$$

$$= (-2x^3 + x^2 + x - 2)\frac{t^2}{2!}$$

$$u_3 = D_t^{-1}[(u_2)_{xx} + u_2 - A_2] = D_t^{-1}[(u_2)_{xx} + u_2 - 4u_0 u_2 - 2u_1^2]$$

$$u_3 = D_t^{-1}[(-12x + 2)\frac{t^2}{2!} + (-2x^3 + x^2 + x - 2)\frac{t^2}{2!} - 4x(-2x^3 + x^2 + x - 2)\frac{t^2}{2!} - 2(x - x^2)^2 t^2]$$

$$u_3 = [(-12x + 2) + (-2x^3 + x^2 + x - 2) - 4x(-2x^3 + x^2 + x - 2) - 4(x - x^2)^2]\frac{t^3}{3!}$$

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$$u(x,t) = u_0 + u_1 + u_2 + u_3 + \cdots$$

$$\cong x + (x - x^2)t + (-2x^3 + x^2 + x - 2)\frac{t^2}{2!} + ((-12x + 2)$$

$$+ (-2x^3 + x^2 + x - 2) - 4x(-2x^3 + x^2 + x - 2) - 4(x - x^2)^2)\frac{t^3}{3!}$$

Second with-fractional, consider $\alpha = 0.5$, $k = 1$ and $v = 1$,

$$u(x,t) - u(x,0) = D_t^{-0.5}(u_{xx} + u - u^2)$$

Define the ADM

$$\sum_{m=0}^{\infty} u_m = u(x,0) + D_t^{-0.5}[(\sum_{m=0}^{\infty} u_m)_{xx} + \sum_{m=0}^{\infty} u_m - \sum_{m=0}^{\infty} A_m]$$

$$A_m = \frac{1}{m!}\frac{d^m}{d\lambda^m}\left[\left(\sum_{j=0}^{m} \lambda^j\, u_j\right)^2\right]_{\lambda=0}$$

Suppose $u_0 = u(x,0) = x$

$$u_1 = D_t^{-0.5}[(u_0)_{xx} - A_0] = D_t^{-0.5}[(u_0)_{xx} + u_0 - u_0^2]$$

$$= D_t^{-0.5}[0 + x - x^2] = 2(x - x^2)\sqrt{\frac{t}{\pi}}$$

$$u_2 = D_t^{-0.5}[(u_1)_{xx} + u_1 - A_1] = D_t^{-0.5}[(u_1)_{xx} + u_1 - 2u_0u_1]$$

$$u_2 = D_t^{-0.5}\left[-4\sqrt{\frac{t}{\pi}} + 2(x - x^2)\sqrt{\frac{t}{\pi}} - 4x(x - x^2)\sqrt{\frac{t}{\pi}}\right]$$

$$u_2 = 2(-4 + 2x - 6x^2 + 4x^3)t$$

$$u_3 = D_t^{-0.5}[(u_2)_{xx} + u_2 - A_2] = D_t^{-0.5}[(u_2)_{xx} + u_2 - A_2]$$

$$u_3 = D_t^{-0.5}[(u_2)_{xx} + u_2 - 4u_0u_2 - 2u_1^2]$$

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$$u(x,t) = u_0 + u_1 + u_2 + u_3 + \cdots \cong x + 2(x - x^2)\sqrt{\frac{t}{\pi}} + 2(-4 + 2x - 6x^2 + 4x^3)t$$

Table 3: The result when time is 0.001, the length is 10, $k = 1$ and $v = 1$

x	$\alpha = 1$	$\alpha = 0.5$
0.0000	-0.0000	-0.0080
1.0000	1.0000	0.9920
2.0000	1.9980	1.9446
3.0000	2.9940	2.8979
4.0000	3.9879	3.8998
5.0000	4.9799	4.9984
6.0000	5.9698	6.2415
7.0000	6.9577	7.6773
8.0000	7.9435	9.3538
9.0000	8.9273	11.3189
10.0000	9.9091	13.6206

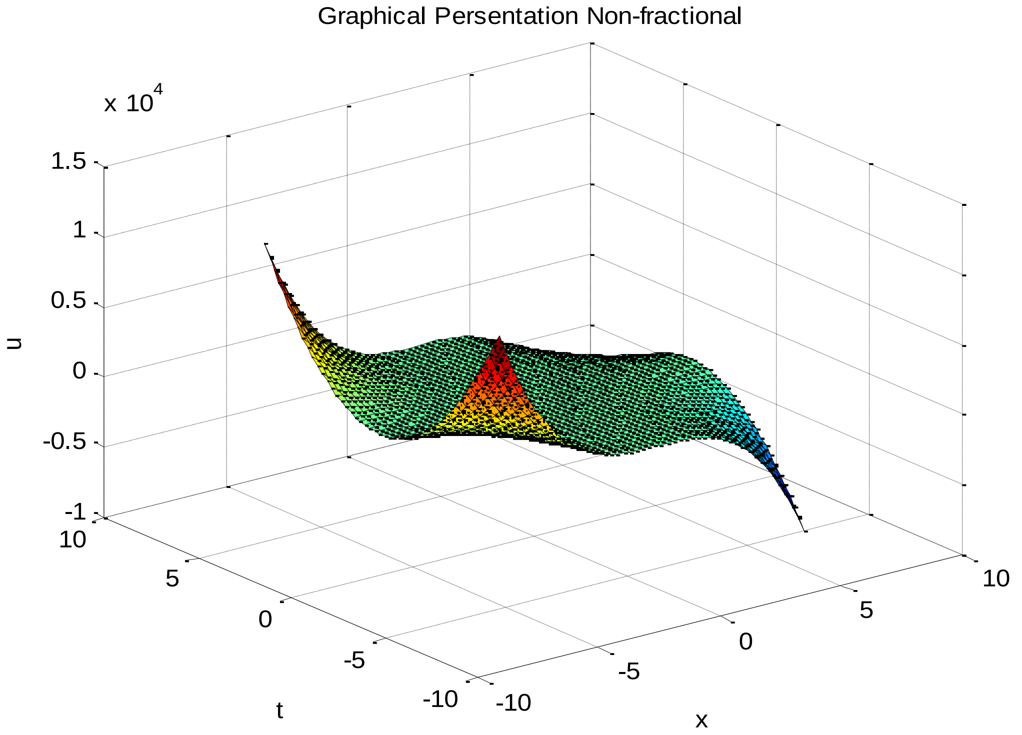

Figure 7: Graphical Presentation Non-Fractional of Fisher's Equation

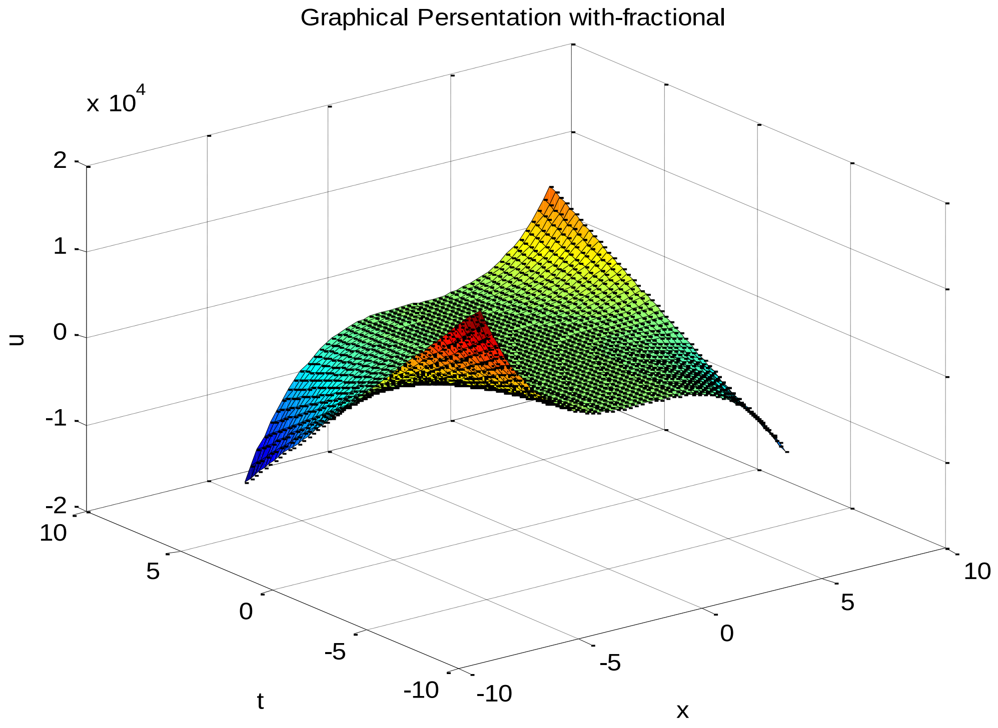

Figure 8: Graphical Presentation with Fractional of Fisher's Equation

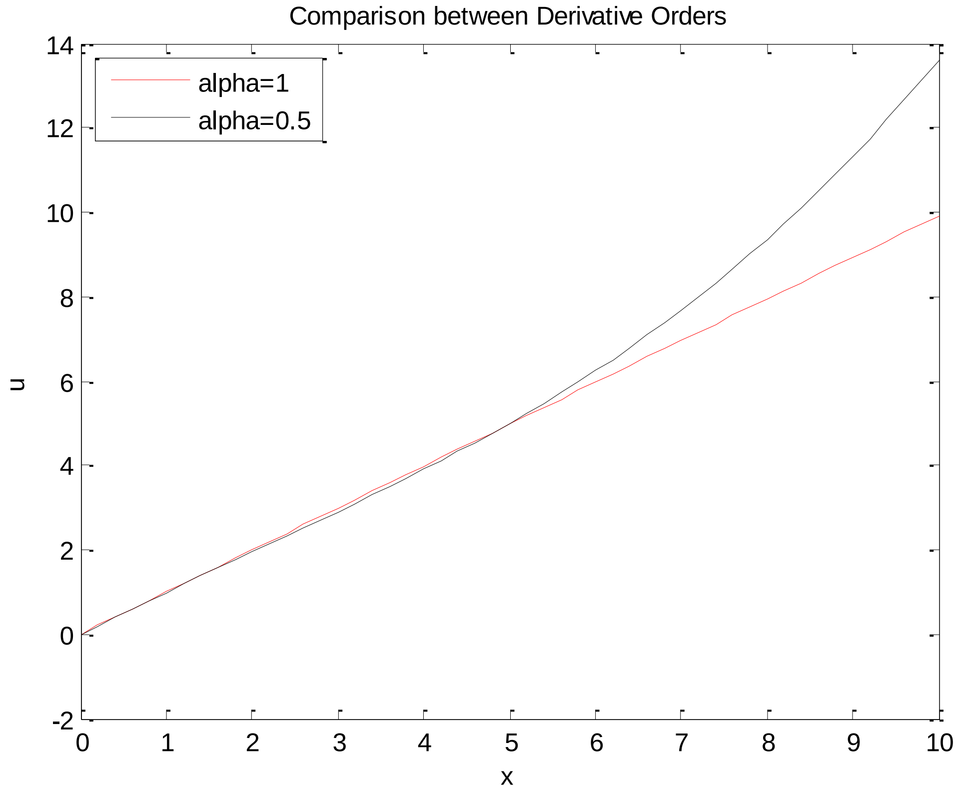

Figure 9: Comparison between derivative Orders of Fisher's Equations

Example 4. Burgers-Fisher equation

$$D_t^{\alpha} u = v u_{xx} + u u_x + k u \left(1 - \frac{u}{k}\right) \ , \qquad k > 0 \ \& \ v > 0, t > 0, \ \ 0 < \alpha \leq 1 \qquad (24)$$

$$\text{IC:} \quad u(x, 0) = x \quad , \quad 0 < x < 10$$

Integration both sides, obtain

$$D_t^{-\alpha} D_t^{\alpha} u = D_t^{-\alpha}(vu_{xx} + uu_x + ku\left(1 - \frac{u}{k}\right))$$

$$u(x,t) - \sum_{k=0}^{n-1} \frac{t^k}{k!} \frac{\partial^k u(x,0)}{\partial t^k} = D_t^{-\alpha}(vu_{xx} + uu_x + ku\left(1 - \frac{u}{k}\right))$$

First Non-fractional, consider $\alpha = 1$, $k = 1$ and $v = 1$,

$$u(x,t) - u(x,0) = D_t^{-1}(u_{xx} + u + uu_x - u^2)$$

Define the ADM

$$\sum_{m=0}^{\infty} u_m = u(x,0) + D_t^{-1}[(\sum_{m=0}^{\infty} u_m)_{xx} + \sum_{m=0}^{\infty} u_m + \sum_{m=0}^{\infty} A_m]$$

$$A_m = \frac{1}{m!} \frac{d^m}{d\lambda^m} \left[\left(\sum_{j=0}^{m} \lambda^j\, u_j \right) \frac{\partial}{\partial x} \left(\sum_{j=0}^{m} \lambda^j\, u_j \right) - \left(\sum_{j=0}^{m} \lambda^j\, u_j \right)^2 \right]_{\lambda=0}$$

Suppose

$$u_0 = u(x,0) = x$$

$$u_1 = D_t^{-1}[(u_0)_{xx} + u_0 + A_0] = D_t^{-1}[(u_0)_{xx} + u_0 + u_0(u_0)_x - u_0^2]$$

$$= D_t^{-1}[0 + x + x - x^2] = (2x - x^2)t$$

$$u_2 = D_t^{-1}[(u_1)_{xx} + u_1 + A_1] = D_t^{-1}[(u_1)_{xx} + u_1 + u_0(u_1)_x + u_1(u_0)_x - 2u_0 u_1]$$

$$u_2 = D_t^{-1}[-2t + (2x - x^2)t + x(2 - 2x)t + (2x - x^2)t - 2x(2x - x^2)t]$$

$$= 0.5(-2 + (2x - x^2) + x(2 - 2x) + (2x - x^2) - 2x(2x - x^2))t^2$$

$$u_3 = D_t^{-1}[(u_2)_{xx} + u_2 + A_2]$$

$$u_3 = D_t^{-1}[(u_2)_{xx} + u_2 + u_0(u_2)_x + u_1(u_1)_x + u_2(u_0)_x - 4u_0 u_2 - 2u_1^2]$$

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$$u(x,t) = u_0 + u_1 + u_2 + u_3 + \cdots$$

$$\cong x + (2x - x^2)t + 0.5(-2 + 2(2x - x^2) + x(2 - 2x) - 2x(2x - x^2))t^2$$

Second with-fractional, consider $\alpha = 0.5$ $k = 1$ and $v = 1$,

$$u(x,t) - u(x,0) = D_t^{-0.5}(u_{xx} + u + uu_x - u^2)$$

Define the ADM

$$\sum_{m=0}^{\infty} u_m = u(x,0) + D_t^{-0.5}[(\sum_{m=0}^{\infty} u_m)_{xx} + \sum_{m=0}^{\infty} u_m + \sum_{m=0}^{\infty} A_m]$$

$$A_m = \frac{1}{m!}\frac{d^m}{d\lambda^m}\left[\left(\sum_{j=0}^{m} \lambda^j\, u_j\right)\frac{\partial}{\partial x}\left(\sum_{j=0}^{m} \lambda^j\, u_j\right) - \left(\sum_{j=0}^{m} \lambda^j\, u_j\right)^2\right]_{\lambda=0}$$

Suppose

$$u_0 = u(x,0) = x$$

$$u_1 = D_t^{-0.5}[(u_0)_{xx} + u_0 + A_0] = D_t^{-0.5}[(u_0)_{xx} + u_0 + u_0(u_0)_x - u_0^2]$$

$$= D_t^{-0.5}[0 + x + x - x^2] = 2(2x - x^2)\sqrt{\frac{t}{\pi}}$$

$$u_2 = D_t^{-0.5}[(u_1)_{xx} + u_1 + A_1] = D_t^{-0.5}[(u_1)_{xx} + u_1 + u_0(u_1)_x + u_1(u_0)_x - 2u_0u_1]$$

$$u_2 = D_t^{-0.5}\left[-4\sqrt{\frac{t}{\pi}} + 4(2x - x^2)\sqrt{\frac{t}{\pi}} + x(4 - 4x)\sqrt{\frac{t}{\pi}} + -4x(2x - x^2)\sqrt{\frac{t}{\pi}}\right]$$

$$= 0.5(-4 + 4(2x - x^2) + x(4 - 4x) - 4x(2x - x^2))t$$

$$u_3 = D_t^{-0.5}[(u_2)_{xx} + u_2 + A_2]$$

$$u_3 = D_t^{-0.5}[(u_2)_{xx} + u_2 + u_0(u_2)_x + u_1(u_1)_x + u_2(u_0)_x - 4u_0u_2 - 2u_1^2]$$

.

.

.

$$u(x,t) = u_0 + u_1 + u_2 + u_3 + \cdots \cong x + 2(2x - x^2)\sqrt{\frac{t}{\pi}} + 2(x^3 - 4x^2 + 3x - 1)t$$

Table 4: The result when time is 0.001, the length is 10, $k = 1$ and $v = 1$

x	$\alpha = 1$	$\alpha = 0.5$
0.0000	**-0.0000**	**-0.0020**
1.0000	**1.0010**	**1.0497**
2.0000	**2.0000**	**2.0580**
3.0000	**2.9970**	**3.0350**
4.0000	**3.9920**	**3.9925**
5.0000	**4.9850**	**4.9428**
6.0000	**5.9761**	**5.8976**
7.0000	**6.9652**	**6.8691**
8.0000	**7.9523**	**7.8692**
9.0000	**8.9374**	**8.9100**
10.0000	**9.9206**	**10.0034**

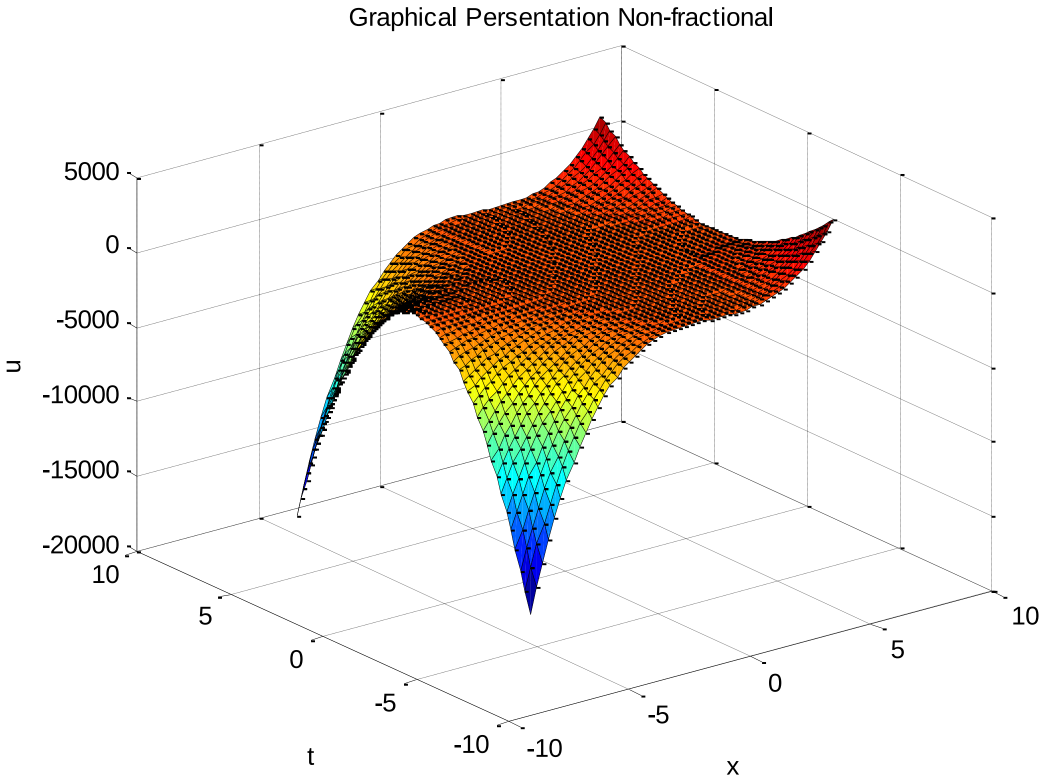

Figure 10: Graphical Presentation Non-Fractional of Burgers-Fisher Equation

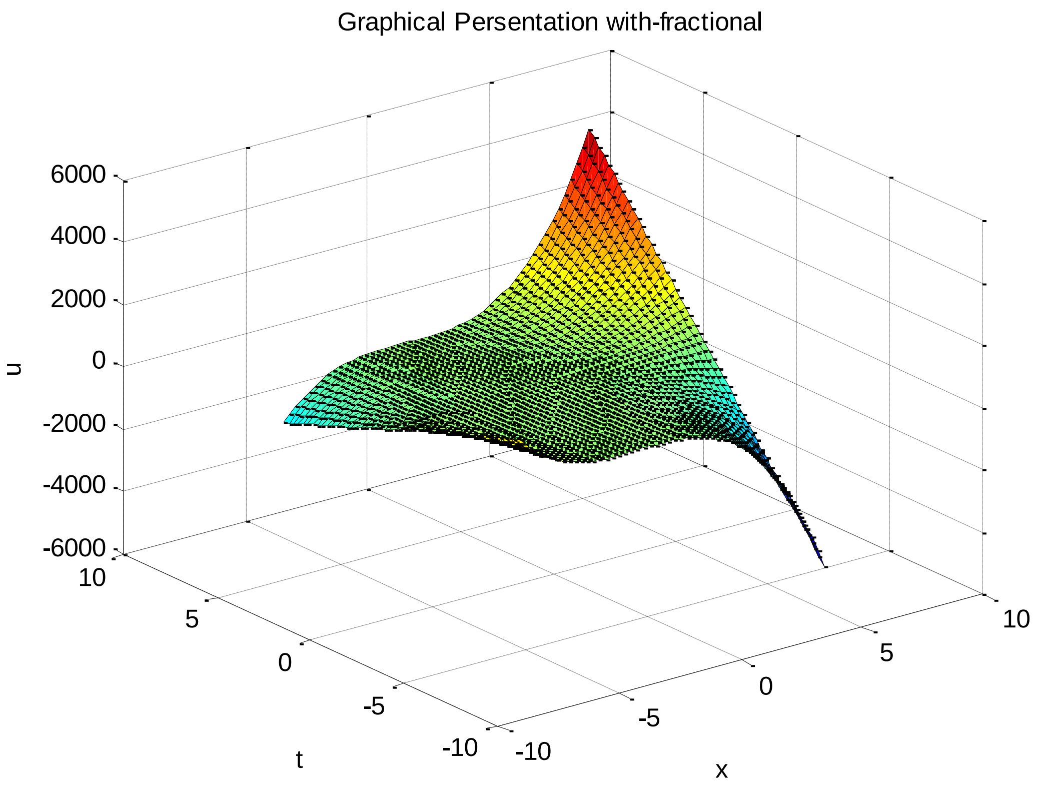

Figure 11: Graphical Presentation with Fractional of Burgers-Fisher Equation

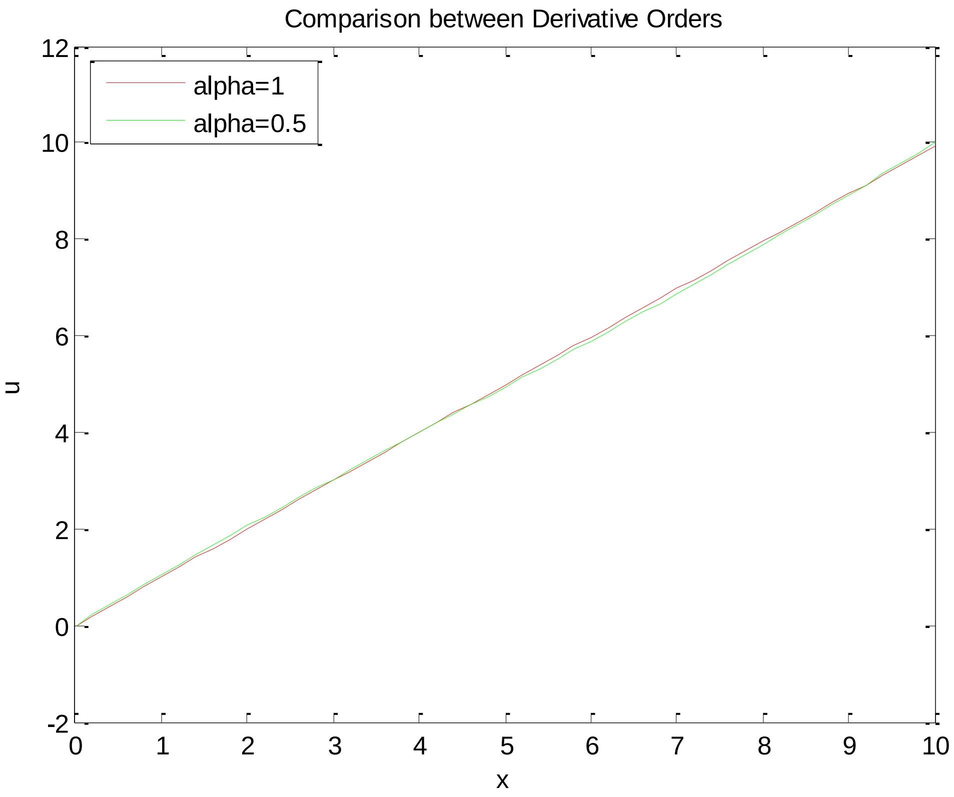

Figure 12: Comparison between derivative Orders of Burgers-Fisher Equation

4 Conclusion

In this paper, the ADM has been successfully applied to obtain the approximation solutions for solving fractional diffusion equations. The solutions are obtained in the form of infinite series; it is easy to see that ADM is powerful mathematical tool for solving different kinds of linear and/or nonlinear fractional partial differential equations arising in different fields of engineering and sciences.

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