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Use the Plasma Equation to Find the Statistical Distribution Laws of Unbalanced Systems

Abdelnabi Ali Elamin[1], Mubarak Dirar Abd-Alla Yagoub[2];

1 Department of physics, Faculty of Science and Technology, Omdurman Islamic University, Sudan
2Department of physics, college of Science Sudan University of science and technology, Khartoum, Sudan.

[*]Corresponding author Email address: *aealamain2016@oiu.edu.sd*

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Abstract

The plasma equation of motion of particles in the presence of afield potential per particle and a pressures force beside a resistive force have been used to find a useful expression of thermal pressure and non-thermal pressure. Another expression of ordinary Maxwell-Boltzmann distribution and Maxwell-Boltzmann distribution with stands for the kinetic energy is derived from the plasma equation with respect to x due to the potential changes only and due to potential changes with change in the density of the number of particles respectively. For non-uniform temperature systems, and non-uniform potential energy per particle, the statistical distribution law is described. This relation is different from where the temperature is assumed to be uniform when the thermal pressure changes due to the temperature change. The statistical distribution law is described when the thermal pressure change due to the temperature change. This relation is different from where the thermal pressure changes due to the change of both particle number density and temperature in this case the plasma equation. Also Statistical Distribution Law from the Plasma Equation in the Presence of Friction has been derived.
Keywords: Plasma Equation, Statistical, Distribution Laws, Unbalanced Systems.

1. Introduction

Statistical physics is one of the most important branches of physics that deals with the physical systems that have very large number of particles. It is concerned with relating the dynamical properties of these individual particles to the equilibrium properties of the whole system [1, 2, 3]. The fundamentals of statistical physics were well established at the end of nineteenth century by Maxwell, Boltzmann and Gibbs and completed by Einstein at 1950 [4, 5]. The foundation of statistical physics is started by Boltzmann who promotes the kinetics theory of gases with the aid of the notion of statistical weight. Later on Gibbs introduce the concept of statistical ensembles to widen the scope of statistical physics.

The laws of statistical physics are now widely used in wide variety of industrial applications. It is well known to scientists that the notion of stimulated emission, which lead to laser discovery, was barn when the laws of statistical physics are utilized to describe absorption and emission processes [6, 7] statistical physics is also widely used in nano science and doping processes to estimate the number of particles on molecules in any micro structure [8, 9, 10]. It is also widely used to describe the thermodynamic behavior of superconductors [11, 12, 13].

Ordinary statistical laws can successfully describe a wide variety of physical phenomena. But unfortunately it is not capable of describing some physical phenomena associated with superconductors, like the specific heat capacity. Move over, it cannot account for the effect of magnetic energy. For instance, if the material is magnetized, its magnetization does not change the potential and kinetic energy. This means that it should contribute to the internal energy. But, the ordinary statistical laws, internal energy and have no room for magnetic energy or other internal energies. This work is concerned with deriving statistical laws from plasma equations.

2. Ordinary Plasma Equation

The plasma equation of motion of particles in the presence of afield potential per particle V and a pressures force P beside a resistive force F_r is given by;

$$nm\frac{dv}{dt} = -\nabla P - F_r - \nabla nV \qquad (1)$$

Where n, m stands for particle number density, and particle mass respectively. Considering the motion to be in one dimension along the x-axis the equation of motion becomes;

$$nm\frac{dv}{dx}.\frac{dx}{dt} = -\frac{dP}{dx} - \frac{d(nV)}{dx} - F_r \qquad (.2)$$

$$nmv\frac{dv}{dx} = -\frac{dP}{dx} - \frac{d(nV)}{dx} - F_r$$

$$n\frac{d\left(\frac{1}{2}mv^2\right)}{dx} = -\frac{dP}{dx} - \frac{d(nV)}{dx} - F_r \qquad (3)$$

The term T stands for the kinetic energy at a single particle and can be written as;

$$T = \frac{1}{2}mv^2 = E_o \qquad (4)$$

The pressure P can also split in to thermal Pt and non-thermal Po to be in the form;

$$P = P_t + P_o = nkT + nP_p \qquad (5)$$

Where Pp in the non-thermal pressure for one particle.

3. Plasma Statistical Equation in the Presence of Potential Field Only

When the potential is only present beside the thermal pressure term the equation of motion (3) reads;

$$n\frac{dE_o}{dx} = -\frac{d(nV)}{dx} - \frac{d(nkT)}{dx} \qquad (6)$$

if one assumes Pt to change with (x) due to the change of (n) only then equation (6) reduces to;

$$n\frac{dE_o}{dx} = -kT\frac{dn}{dx} - \frac{d(nV)}{dx} \qquad (7)$$

The temperature here is assumed to be uniform; here one has two cases either $V_T = nV$ changes with respect to x due to the change of V only. In this case equation (7) reads;

$$n\frac{dE_o}{dx} = -kT\frac{dn}{dx} - n\frac{dV}{dx} \qquad (8)$$

$$n\frac{d(E_o+V)}{dx} = -kT\frac{dn}{dx} \qquad (9)$$

The total energy is given by;

$$E = T + V = E_o + V \qquad (10)$$

Therefore (9) becomes;

$$n\frac{dE}{dx} = -kT\frac{dn}{dx}$$

$$ndE = -kTdn$$

Integration both sides yields;

$$-\int\frac{dE}{kT} = \int\frac{dn}{n}$$

$$\ln n = -\frac{E}{kT} + C_o$$

$$n = C\, e^{\frac{-E}{kT}} \qquad (11)$$

This is the ordinary Maxwell-Boltzmann distribution. But if VT changes due to change of (n) only, then equation (7) reads;

$$n\frac{dE_o}{dx} = -kT\frac{dn}{dx} - V\frac{dn}{dx}$$

$$n\frac{dE_o}{dx} = -(kT + V)\frac{dn}{dx}$$

$$-\frac{dE_o}{(kT+V)} = \frac{dn}{n}$$

Integration both sides yields;

$$\int \frac{dn}{n} = -\int \frac{dE_o}{(kT+V)}$$

$$\ln n = -\frac{E_o}{(kT+V)} + C_o$$

$$n = C\, e^{\frac{-E_o}{(kT+V)}} \qquad (12)$$

The energy E_0 here stands for the kinetic energy only as shown by equation (9).

4. Plasma Statistical Equation When Thermal Pressure Changes Due to the Temperature Change

When the thermal pressure change due to the temperature change;

$$\frac{dP_t}{dx} = n\frac{d(kT)}{dx} \qquad (13)$$

In this case the plasma equation (3) in the absence of a resistive force is given by;

$$n\frac{dE_o}{dx} = -\frac{dP_t}{dx} - \frac{d(nV)}{dx}$$

$$n\frac{dE_o}{dx} = -n\frac{d(kT)}{dx} - \frac{d(nV)}{dx} \qquad (14)$$

Where the pressure here is assumed to be due to the thermal pressure only. If the total potential VT is assumed to be related to the rate of change at V only, I.e.

$$\frac{dV_T}{dx} = \frac{d(nV)}{dx} = n\frac{dV}{dx} \qquad (15)$$

In this case equation (14) reads;

$$n\frac{dE_o}{dx} = -n\frac{d(kT)}{dx} - n\frac{dV}{dx}$$

$$\int dE_o = -\int d(kT) - \int dV$$

$$E_o = -kT - V + C_o$$

Thus;

$$C_o = E_o + kT + V$$

One can easily deduce that Co is equal to the total energy E, I.e.

$$E = E_o + V + kT \qquad (16)$$

I.e. the total energy is equal to kinetic energy Eo beside potential energy V and thermal energy KT. But if VT change due to the rate of change of n only, I.e.

$$\frac{dV_T}{dx} = V\frac{dn}{dx} \qquad (17)$$

Equation (14) thus reads;

$$n\frac{dE_o}{dx} = -n\frac{d(kT)}{dx} - V\frac{dn}{dx}$$

$$n(dE_o + dkT) = -Vdn$$

$$\int \frac{dn}{n} = -\int \frac{(dE_o+dkT)}{V}$$

$$\ln n = -\frac{(E_o+kT)}{V} + C_o$$

$$n = C\, e^{\frac{-(E_o+kT)}{V}} \qquad (18)$$

If the change of (VT) with respect to (x) is due to the change of both (n) and (V) with respect to (x);

$$\frac{dV_T}{dx} = \frac{d(nV)}{dx} = n\frac{dV}{dx} + V\frac{dn}{dx} \qquad (19)$$

Inserting (16) in (14) yields;

$$n\frac{dE_o}{dx} = -n\frac{d(kT)}{dx} - n\frac{dV}{dx} - V\frac{dn}{dx}$$

$$n\, d(E_o + kT + V) = -Vdn$$

$$\int \frac{dn}{n} = -\int \frac{d(E_o+kT+V)}{V}$$

$$\ln n = -\frac{(E_o+kT+V)}{V} + C_o$$

$$n = Ce^{\frac{-(E_o+kT+V)}{V}} \qquad (20)$$

Thus for non-uniform temperature systems, and non-uniform potential energy per particle, the statistical distribution law is described by (20). This relation is different from that obtained in (9), where the temperature is assumed to be uniform.

5. Plasma Statistical Equation When Thermal Pressure Change Due to the Change of Both (n) and (T)

When the thermal pressure changes due to the change of both (n) and (T), in this case the plasma equation (14) is given by;

$$n\frac{dE_o}{dx} = -n\frac{d(kT)}{dx} - kT\frac{dn}{dx} - \frac{d(nV)}{dx} \qquad (21)$$

If the total potential (VT) is assumed to be related to the rate of change of (V) only, I.e

$$\frac{dV_T}{dx} = \frac{d(nV)}{dx} = n\frac{dV}{dx} \qquad (22)$$

In this case equation (21) reads;

$$n\frac{dE_o}{dx} = -n\frac{d(kT)}{dx} - kT\frac{dn}{dx} - n\frac{dV}{dx}$$

$$n(dE_o + d(kT) + dV) = -kTdn$$

$$\int \frac{dn}{n} = -\int \frac{(dE_o+d(kT)+dV)}{kT}$$

$$\ln n = -\frac{(E_o+kT+V)}{kT} + C_o$$

$$n = C\, e^{\frac{-(E_o+kT+V)}{kT}} \qquad (23)$$

But if VT change due to the rate of change on n only, I.e.

$$\frac{dV_T}{dx} = \frac{d(nV)}{dx} = V\frac{dn}{dx} \qquad (24)$$

In this case equation (21) reads;

$$n\frac{dE_o}{dx} = -n\frac{d(kT)}{dx} - kT\frac{dn}{dx} - V\frac{dn}{dx}$$

$$n\big(dE_o + d(kT)\big) = -(kT + V)dn$$

$$\frac{dn}{n} = -\frac{\big(dE_o+d(kT)\big)}{(kT+V)}$$

$$\int \frac{dn}{n} = -\int \frac{(dE_o+dkT)}{(kT+V)}$$

$$\ln n = -\frac{(E_o+kT)}{(kT+V)} + C_o$$

$$n = C\, e^{\frac{-(E_o+kT)}{(kT+V)}} \qquad (25)$$

If the change of VT with respect to x is due to the change of both n and V with respect to x, then;

$$\frac{dV_T}{dx} = \frac{d(nV)}{dx} = n\frac{dV}{dx} + V\frac{dn}{dx} \qquad (26)$$

Inserting (26) in (21) yields;

$$n\frac{dE_o}{dx} = -n\frac{d(kT)}{dx} - kT\frac{dn}{dx} - n\frac{dV}{dx} - V\frac{dn}{dx}$$

$$n(dE_o + dV + dkT) = -(kT + V)dn$$

$$\int\frac{dn}{n} = -\int\frac{(dE_o+dV+dkT)}{(kT+V)}$$

$$\ln n = -\frac{(E_o+ kT +V)}{(kT+ V)} + C_o$$

$$n = C\, e^{\frac{-(E_o+ kT+V)}{(kT+V)}} \qquad (27)$$

6. Statistical Distribution Law From the Plasma Equation in the Presence of Friction

The plasma equation (3) can be written when only thermal pressure and frictional force acts in the form;

$$n\frac{dE_o}{dx} = -kT\frac{dn}{dx} - nF_r \qquad (28)$$

Where Fr is the frictional force per particle.

Multiplying both sides by dx, one gets;

$$n\, dE_o = -kTdn - nF_r.dx \qquad (29)$$

$$n(dE_o + F_r.dx) = -kTdn$$

$$\frac{dn}{n} = -\frac{1}{kT}(dE_o + F_r.dx)$$

$$\int\frac{dn}{n} = -\frac{1}{kT}\left(\int dE_o + \int F_r.dx\right)$$

$$\ln n = -\frac{1}{kT}\left(E_o + \int F_r.dx\right) + C_o$$

$$n = C\, e^{\frac{-(E_o+\int F_r.dx)}{kT}} \qquad (30)$$

Thus the number of particles n having energy Eo and subjected to a frictional force Fr is given by (30).

7. Discussion

According to the plasma equation the ordinary statistical distributions arises from the fact that the pressure P changes due to the change of number of particles only, while the temperature T remains uniform as shown in section (3) by equations (.6,7,11). If the total potential V is also spatially vary due to the variation of the density of particles spatially, the distribution function (12) does no longer resemble Maxwell distribution. This is since the denominator consists of kT beside the potential per particle V, this resembles the work done by some authors [see section (3.7)]. In the case when the pressure change due to temperature spatial change, the numerator consists of thermal energy beside kinetic and potential energy per particle V, while the denominator consists of the potential energy per particle only as shown by relation (20).

If the pressure and potential changes due to the change of T, V, n, the terms kT and V both appears in the denominator and numerator. The term E_0 which stands for the kinetic energy appear only in the numerator. This indicates that temperature stands only for random kinetic motion, while the drift velocity driven by forces appears as aspirate term as shown in equation (27).

The effect of friction on the distribution of particles is shown in section (4.6). The number of particles n is affected directly by friction through the exponential term as shown by equation (30).

``8. Conclusion

The statistical distribution laws for non-equilibrium systems and frictional medium can be obtained from plasma equation. One can deduce the Gibbs distribution, beside other distributions in which the systems are not in thermal equilibrium. The diffusion current resulting from concentration and thermal gradient emerges naturally. Such distribution can explain the lattice specific heat capacity of superconductors. The frictional system can also be described within the frame work of this system.

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