بسم الله الرحمن الرحيم
Omdurman Islamic University Journal of Applied Sciences (OIUJ)-(ASC)
مجلة جامعة أم درمان الاسلامية للعلوم التطبيقية
2025; 21(3)65-87
https://journal.oiu.edu.sd/index.php/oiuj
https://doi.org/ 10.52981/oiujas.v21i3.3397
ISSN: 5361-1858

Environmental-Adjusted Operational Efficiency Analysis of Insurance Companies: An Application of the Three-Stage Data Envelopment Analysis Model

FADWA ABD ALLA IBRAHEM AHMED[1], KHALID RAHAMTALLA KHEDIR GENAWI[2]

[1]Department of Mathematics – Faculty of Science and Technology – Omdurman Islamic University
[2]Department of Statistics – Faculty of Sciences – Sudan University of Science and Technology

E. mail: fadwaabdallah@gmail.com[1] , genawi@gmail.com[2]

To cite this article:

FADWA ABD ALLA IBRAHEM AHMED, KHALID RAHAMTALLA KHEDIR GENAW Environmental-Adjusted Operational Efficiency Analysis of Insurance Companies: An Application of the Three-Stage Data Envelopment Analysis Model , Omdurman Islamic University Journal, Sudan

ISSN: 5361-1858

https://doi.org/ 10.52981/oiujas.v21i3.3397

Abstract

companies from 2012 to 2020 using the Three-Stage Data Envelopment Analysis (DEA) model. The first stage applies an input-oriented DEA model to estimate efficiency scores. In the second stage, Stochastic Frontier Analysis (SFA) is employed to separate inefficiency from environmental factors and random noise, focusing on company age and type. The third stage uses adjusted inputs to reassess efficiency, providing more accurate and fair estimates. The results show th 10.52981/oiujas.v21i3.3397 at company age significantly affects efficiency, while company type has limited influence. Overall, efficiency scores improved after adjustment, highlighting the value of incorporating environmental variables in performance evaluations. The study recommends adopting the Three-Stage DEA model for more reliable efficiency assessments in the insurance sector.

Keywords: Operational efficiency, Data Envelopment Analysis, Stochastic Frontier Analysis, Environmental factors, Insurance companies, Sudan.

1. Introduction

The evaluation of producer performance is a fundamental concern in operations research and efficiency analysis. Traditionally, producers are assessed based on their ability to optimize the use of inputs to generate a given level of outputs, or to maximize outputs from a fixed set of inputs, relative to comparable decision-making units (DMUs). In this context, Data Envelopment Analysis (DEA), introduced by (Charnes et al.,1978), has emerged as a widely accepted non-parametric technique for benchmarking relative efficiency.

However, observed performance is rarely the result of managerial effort alone. It is also shaped by exogenous environmental conditions—such as market structure, regulatory settings, and organizational context—and random shocks or unobserved influences, often referred to as statistical noise. Disentangling these three sources of variation—managerial inefficiency, environmental effects, and random noise—is essential for deriving fair and accurate efficiency estimates.

While conventional DEA models offer robust efficiency assessments, they are inherently deterministic and therefore unable to isolate the influence of randomness or accommodate environmental heterogeneity. Previous attempts to address this limitation have led to the development of single-stage and two-stage DEA models. Single-stage models incorporate environmental variables directly into the efficiency analysis, but often suffer from specification limitations and ignore statistical noise. Two-stage models introduce environmental effects through post-DEA regression analysis; however, unless stochastic methods are used in the second stage, they too remain deterministic.

In response to these methodological constraints, this study proposes a Three-Stage DEA model that integrates Stochastic Frontier Analysis (SFA) in the second stage. This framework allows for a complete decomposition of performance variation into its three fundamental components. The model proceeds as follows:

- **Stage 1:** Efficiency is initially estimated using standard DEA based on inputs and outputs alone.
- **Stage 2:** Using SFA, inefficiency scores are regressed on environmental variables to isolate the effects of external conditions and statistical noise.
- **Stage 3:** The original input data are adjusted based on the estimates from Stage 2, and DEA is reapplied to obtain refined efficiency scores.

The empirical section of this study applies the proposed model to a sample of 12 Sudanese property insurance companies, covering the period 2012–2020. The aim is to derive efficiency scores that more accurately reflect managerial performance, free from

external and random distortions. The study concludes with recommendations for both practitioners and policymakers to enhance efficiency measurement and resource allocation within the insurance sector.

2. Literature Review

The Three-Stage DEA model has received increasing attention in empirical studies due to its ability to distinguish between true technical efficiency and the effects of environmental factors and random noise. The integration of the Stochastic Frontier Analysis (SFA) model in the second stage has proven to be an effective tool for improving estimation accuracy.

1. In this context, (Chen & Yu and Hu,2011) investigated the efficiency of Chinese banks using the Three-Stage DEA model. Their findings indicated that environmental factors such as bank size and geographic location significantly influenced the apparent (unadjusted) efficiency. In the field of higher education efficiency, (Sun & Lu,2015) applied the model to Chinese universities and found that government support and research environment had a notable impact on apparent efficiency before adjustment using SFA.
2. (Li & Lin,2017) conducted a study on Chinese insurance companies and found that the second stage helped isolate the impact of regional regulations, allowing for a more accurate assessment of true efficiency. Similarly, (Fukuyama & Matousek,2017) applied the model to European banks in the post-financial crisis period, emphasizing the importance of SFA in revealing diverse market effects. In the same vein, (Wang & Zhou,2018) applied the model to solid waste management companies and demonstrated that environmental differences, such as population size and service scope, influenced unadjusted efficiency scores.
3. (Liu et al.,2020) utilized the model to analyze the efficiency of rural hospitals and found that the second stage reduced the impact of disparities in government funding and infrastructure. (Yin & Zhang,2020) studied urban transportation firms using the same model and emphasized the necessity of the second stage in identifying inefficiency sources that stem from environmental factors rather than operational shortcomings.
4. On another front, a study by (Zhou et al.,2021) analyzed data from 65 publicly listed e-commerce firms in China over the period 2015–2019. Their results showed that the SFA stage revealed considerable variations in scale efficiency after adjusting operational efficiency, reflecting a heterogeneous effect of external environmental factors.
5. In the area of carbon efficiency, (Wu,2023) analyzed data from nine cities in China's Fujian Province during 2015–2020 and concluded that the three-stage model contributed to uncovering latent potentials for emission reduction through improvements in pure technical efficiency. Similarly, (Wu & Chen,2023)

highlighted the effectiveness of the model in evaluating green transportation and energy companies, as it helped reveal the influence of external markets and competitive environments.

3. Clarify the Research Gap

Although the Three-Stage DEA model has gained traction in international research, especially when combined with Stochastic Frontier Analysis (SFA), there is a complete absence of such integrated applications in the Sudanese insurance sector. Existing local studies on insurance efficiency have primarily relied on traditional DEA or basic financial ratio analysis, without addressing the influence of environmental factors or random noise. To the best of our knowledge, this is the first study in Sudan to employ the full Three-Stage DEA model alongside SFA in evaluating the performance of insurance firms. This unique methodological integration aims to fill a significant research gap by offering a more accurate, fair, and context-sensitive assessment of operational efficiency in the Sudanese insurance industry.

4. Research Objectives

The primary aim of this study is to evaluate and improve the measurement of operational efficiency among Sudanese insurance companies by applying a robust three-stage DEA model. The specific objectives are as follows:

1. To estimate the relative operational efficiency of Sudanese insurance companies during the period 2012–2020 using an input-oriented Data Envelopment Analysis (DEA) approach.
2. To examine the influence of environmental factors, specifically company type (public vs. private) and company age, on efficiency levels by incorporating these variables into the second stage of the model using Stochastic Frontier Analysis (SFA).
3. To distinguish between managerial inefficiency and statistical noise through SFA, thereby enhancing the precision of the efficiency estimates.
4. To re-calculate efficiency scores in the third stage by adjusting inputs for the effects of environmental conditions and random variation, producing more realistic efficiency measurements.
5. To compare pre- and post-adjustment efficiency scores between the first and third stages in order to assess the extent to which external factors and randomness distort initial DEA results.
6. To propose a more reliable and equitable performance evaluation framework that supports data-informed decision-making and resource optimization in the insurance sector.

5. Research Assumptions

This study is based on the following key assumptions, which underpin the methodological framework and guide the interpretation of the efficiency estimates:

1. The efficiency scores generated in the first stage are subject to distortion due to the presence of environmental influences and random noise, which may bias the initial DEA results.
2. It is assumed that the inefficiency component (u_{ij}) and the random error term (v_{ij}) are statistically separable, enabling a more precise decomposition of performance variation using Stochastic Frontier Analysis (SFA).
3. The input variables—Capital, Total Rights, Net Premiums, and Total Direct Premiums for Takaful—contain noise components that are statistically identifiable and can be adjusted for in the third stage.
4. The study assumes that some variables deviate from normality, as confirmed by the Shapiro–Wilk test, which justifies the application of non-parametric techniques such as DEA within the chosen analytical framework.
5. The average efficiency scores obtained after input adjustment in the third stage provide a more accurate reflection of actual company performance, as they account for environmental and stochastic distortions present in the raw data.
6. Adjusting input variables based on second-stage SFA results leads to more realistic efficiency estimates in the third stage, improving the reliability of the overall evaluation process.

6. Analytical Tools and Statistical Software

The analysis in this study was conducted using R programming language, which enabled the implementation efficiency analysis techniques. Specifically, the study employed:

- Data Envelopment Analysis (DEA) for the initial and final efficiency measurements.
- Stochastic Frontier Analysis (SFA) in the second stage to isolate managerial inefficiency, environmental influences, and statistical noise.

These tools collectively ensured a robust and statistically sound assessment of operational efficiency among the studied insurance firms.

7. Current Approaches for Incorporating Environmental Effects into DEA

Single-stage methods were developed by (Banker & Morey,1986a,1986b) to address non-discretionary environmental variables (such as quasi-fixed inputs or outputs

temporarily constrained by contractual obligations), as well as categorical variables (such as ownership type). The approach to handling non-discretionary variables involves incorporating them along with the inputs and outputs into the DEA model, while restricting optimization to either inputs or outputs only. This method requires a prior understanding of the direction in which each non-discretionary variable affects the performance of the decision-making unit (DMU).

For categorical variables, the comparison set is limited to DMUs within the same category, or a higher or lower category—assuming that these categories are nested. However, this constraint reduces the size of the comparison group for most units, thereby weakening the model's ability to discriminate between efficiency levels. Both of these methods are fully deterministic, and hence, they cannot incorporate the effects of statistical noise in performance evaluations. A detailed critique of these single-stage approaches was presented by (Adolphson et al.,1991).

The typical two-stage approach begins with a first-stage DEA analysis using input and output data, followed by a second-stage regression analysis that explains variations in efficiency scores from the first stage based on a vector of observable environmental variables. This approach was first proposed by (Timmer ,1971). Subsequent studies enhanced the second stage by adopting limited dependent variable regression techniques, since efficiency scores are bounded and often reach the upper limit.

(McCarty & Yaisawar,1993) and (Bhattacharyya et al.,1997) extended this framework by using regression residuals in the second stage to adjust the efficiency estimates from the first stage. (Fried et al.,1993) further expanded the analysis by applying regression to input and output slacks instead of efficiency scores, utilizing Seemingly Unrelated Regression (SUR) techniques.

(Pastor,1995) introduced a novel two-stage approach termed "Double DEA Analysis". In the first stage, DEA was applied using either an input- or output-oriented model, incorporating environmental variables in the data. Then, inputs or outputs were replaced with their radial projections to remove the environmental influence. In the second stage, DEA was re-applied to an extended dataset that included: originally efficient observations, originally inefficient observations, and the radial projections of the inefficient ones. By comparing the efficiency scores of originally inefficient observations with their projections, it was possible to measure the influence of environmental factors on DMU performance.

The two-stage method was also extended by (Fried et al.,1999). Their methodology begins with an initial DEA evaluation, followed by a second-stage Tobit regression to estimate the effects of environmental variables on efficiency scores. In the third stage, the original data are adjusted to reflect these environmental effects, and DEA is

reapplied. The advantage of this approach lies in the stochastic framework used in the second stage, which allows for the isolation of environmental effects. However, a limitation of the method is that the adjustment does not account for statistical noise.

8. The Three-Stage DEA Model

Our approach for refining the evaluation of producer performance from the effects of environmental variables and statistical noise follows a three-stage model. In the first stage, we apply the standard DEA model to input and output data to obtain an initial assessment of producer performance. This initial evaluation does not consider the operational environment or statistical noise.

Therefore, in the second stage, we apply Stochastic Frontier Analysis (SFA) to decompose the variation in first-stage efficiency results into environmental effects, managerial inefficiency, and statistical noise.

In the third stage, we adjust the producers' inputs or outputs based on the environmental effects and statistical noise identified in the second stage. Then, we repeat the first-stage DEA analysis on the adjusted dataset. This final stage provides refined estimates of managerial efficiency, as the influence of environmental factors and statistical noise has been removed through the SFA-based corrections.

8.1 First-Stage*: Baseline Efficiency Assessment Using DEA*

The initial assessment of producer performance is conducted using the traditional oriented DEA model, relying solely on quantitative input and output data.
For a given producer o, the envelope model under the assumption of Variable Returns to Scale (VRS), as proposed by (Banker et al.,1986) is expressed as follows

programming problem:

$$\min_{\Theta, \gamma} (\Theta)$$

subject to $\theta x^{o} \geq X\lambda$

$Y\lambda \geq y^{o}$

$\lambda \geq 0$

$e^{T}\lambda = 1$

where $x \geq 0$ is a producer's $N \times 1$ vector of inputs, $y \geq 0$ is a producer's $M \times 1$ vector of outputs, $X = [x_1,..., x_I]$ is an $N \times I$ matrix of input vectors in the comparison set, $Y = [y_1,..., y_I]$ is an $M \times I$ matrix of output vectors in the comparison set, $\lambda = [\lambda_1,..., \lambda_I]$ is an I

× 1 vector of intensity variables, *e* = [1,..., 1] is an *I* × 1 vector, and there are *I* producers in the comparison set. The data of the producer being evaluated are superscripted

"o", and the problem is solved *I* times, once for each producer in the comparison set.

The optimal solutions to the envelopment problem (1) provide initial performance evalua- tions for each producer, expressed in terms of the optimal values of $\theta \leq 1$ and the nonnegative slacks in the $M + N$ functional constraints.

(continued) Optimal Solutions from Model (1)

The optimal solutions from Model (1) provide an initial evaluation of each producer's performance, expressed through the optimal value $\theta \leq 1$ and the non-negative slacks in the input and output constraints, totaling M+N. However, actual performance may result from a combination of:

1. Managerial inefficiency,
2. Environmental effects,
3. Statistical noise (measurement errors).

Hence, isolating these three components is desirable. However, this cannot be achieved through the above model due to the absence of environmental variables and statistical noise terms.

8.2 Second Stage: Applying the SFA Model to Decompose First-Stage Deviations

In the second stage, a Stochastic Frontier Analysis (SFA) model is used to analyze the residuals from the first-stage DEA. This is done in order to address the potential influence of environmental factors and random fluctuations on producer performance.

In this stage the residuals, the differences between observed input values and the predicted (or expected) inputs are modeled using multivariate regression.

The SFA framework assumes that the residuals consist of three components:

1. A systematic environmental component ($Zi\beta$), representing the structured influence of environmental variables on input use,
2. A random noise term (vi), which captures unpredictable or statistical variations,
3. A non-negative inefficiency term (ui), associated with deviation from the efficient frontier due to managerial inefficiency.

A separate regression model is constructed for each input from the first stage. For each deviation (slack) in the use of input i by producer n, the following SFA model is specified:

$S_{ij} = f^{j}(z_i ; \beta^j) + v_{ij} + u_{ij}$, $i=1,2,\ldots.,I$, $j=1,2,\ldots.,n$ (2)

Where:

- s_{ni} is the slack (residual) of input i for producer j,
- z_i is a vector of observable environmental variables for producer j,
- $f^{i}(z_i ; \beta^j)$ is deterministic feasible slack frontiers with parameter vectors β^j to be estimated and composed error structure $(v_{ji} + u_{ji})$.
- $V_{ji} \sim N(0,\sigma^2)$ represents the random noise,
- $U_{ji} \sim N^{+}(\mu^j,\sigma^2)$ is the non-negative inefficiency term, assumed to follow a truncated-normal or half-normal distribution.

This formulation enables the decomposition of each producer's input inefficiency into environmental influences, random noise, and managerial shortcomings.

8.2.1 Advantages of Incorporating SFA in the Second Stage of the DEA Framework

1. No prior knowledge is required regarding the direction of environmental variable effects.
2. The statistical significance of these effects can be tested using Likelihood Ratio (LR) tests.
3. The model allows for testing the hypothesis of homogeneous managerial efficiency across producers.
4. It accommodates heterogeneous effects of environmental variables, noise, and inefficiency across different input variables.

8.2.2 Input Data Adjustment

After identifying the effects of environment and statistical noise, the input variables are adjusted to create a "level playing field" for all producers. The adjustment aims to eliminate the external influences and noise, enabling a fairer efficiency reassessment in the third stage.

The adjusted input for each producer is computed using the following expression:

$$S_{ij} = f(Z_j, \beta) + v_{ij} - u_{ij} \qquad (3)$$

The slack in the use of input i by unit j is denoted by S_{ij}, where:

- Z_j represents the vector of environmental variables for unit j,
- v_{ij} denotes the random noise component, and
- u_{ij} captures the inefficiency component.

After estimating this equation using Stochastic Frontier Analysis (SFA), the original input data are adjusted to reflect a "fair competitive environment" among the producers. The adjusted inputs are calculated using the following formula:

$$*_{X_{ij}} = X_{ij} + \left[max\left(\hat{}_{ij}u\right) - \hat{}_{ij}u)\right] + max\left(f\left(Z_j\right)\right)) + f(Z_j)] \qquad (4)$$

Where:

- $*_{X_{ij}}$ is the adjusted input i for unit j,
- X_{ij} is the original input value before adjustment,
- $\hat{}_{ij}u$ is the estimated random noise component from the second stage,
- $f\left(Z_j\right)$ is the estimated environmental effect,
- Max(.) denotes the maximum value used as a reference benchmark.

orientation as before (input-oriented or output-oriented), in order to obtain the efficiency scores after correcting for deviations. This new evaluation is referred to as the environmentally-adjusted efficiency.

Substantial Differences in Results

Empirical studies indicate that the efficiency scores obtained in the third stage often differ significantly from those calculated in the first stage. These differences are analytically meaningful, as they reflect the impact of removing environmental effects and statistical noise from the initial performance evaluation.

8.2.3 Estimating the Impact of Statistical Noise in Efficiency Analysis

To estimate the components of the error term, we adopt the method proposed by (Jondrow et al.,1982). This technique allows the separation of the composite error term into its two distinct components:

$$E\hat{} = [v_{ji} + u_{ji}] = \left[v_{ji} + u_{ji}\right] = s_{ji} - z_i\beta\hat{}j - E\hat{}[u_{ji}\backslash v_{ji} + u_{ji}]$$

$$j = 1,\ldots,n \quad , \quad i = 1,\ldots, I \qquad (5)$$

$$E^{\wedge} = [v_{ji} + u_{ji}] = s_{ji} - z_i \beta^{\wedge} j - E^{\wedge}[u_{ji} \backslash v_{ji} + u_{ji}]$$

: Where:

$E^{\wedge} = [v_{ji} + u_{ji}]$ is calculated based on the conditional expectation of the inefficiency component given the estimated total residual.

8.3 Re-estimating Efficiency After Environmental and Statistical Adjustments

In the final stage, the original input values are adjusted based on the results of the second-stage analysis, specifically by removing the influence of environmental factors and random noise components. These adjusted values are then used as new inputs for reapplying the conventional DEA model.

This step levels the playing field by equalizing all decision-making units (DMUs) under the most favorable conditions (i.e., the best observed environment), through compensating for environmental and random inefficiencies.

Subsequently, the pure technical efficiency is recalculated using the adjusted inputs, allowing for a fairer comparison across DMUs, since external, uncontrollable factors have been filtered out.

9. Advantages of the Three-Stage DEA Model

The Three-Stage DEA model offers the following benefits:

- Fairness in evaluation: By eliminating the effects of environmental and stochastic factors, efficiency scores become more objective and equitable.
- Deeper interpretation: The model provides insightful explanations for inefficiency, whether stemming from environmental conditions or internal managerial performance.
- Analytical flexibility: It can be applied across a wide range of sectors, including healthcare, education, insurance, and public services.

However, the model's effectiveness depends on the quality of data and the relevance of selected environmental variables. Additionally, the SFA model assumes a specific statistical distribution, which may limit its accuracy in certain empirical contexts.

10. Analysis Results

10.1 Descriptive Analysis of Inputs, Outputs, and Environmental Variables

The table presents descriptive statistics for a set of variables used in the efficiency analysis of insurance companies. These include first-stage input variables as well as

two environmental variables. The data reveal considerable variation across most variables, reflecting significant disparities in company size and performance levels (**see Appendix Table A1).**

For the input variables, the average capital was approximately 10 million, with a standard deviation of around 10.6 million, indicating the presence of both low-capital and high-capital companies, with the latter reaching up to 100 million. The mean total rights amounted to 27.3 million, with a high standard deviation of 82.5 million, and a maximum value exceeding 851 million, highlighting the presence of companies with a substantial financial base compared to others. Regarding the output variables, the average net premiums reached 73.4 million, with a very high standard deviation of 140.2 million and a maximum value of over 670 million, demonstrating significant differences in insurance activity among the firms. Additionally, some companies recorded zero values for certain outputs, including net premiums and total direct premiums for Takaful, possibly indicating partial or complete suspension in specific insurance lines. In terms of investment deposits in banks, the values ranged from zero to approximately 61 million, with a mean of 5.3 million, reflecting differences in investment strategies. The outward premiums averaged 38.8 million, with a standard deviation of 66.9 million. Profits varied widely, with a maximum of 2.1 billion and a minimum of only 293,000, indicating stark contrasts in profitability across the firms. As for the environmental variables, the average company age was 44.25 years, with a standard deviation of 9.26 years, suggesting that most companies are relatively old, with values ranging from 32 to 63 years. The company type variable, coded as 0 or 1, had a mean of 0.08, indicating a dominance of one category; in this case, only one public company was included in the sample, with the rest being private These results highlight the substantial variability in resources and financial outcomes among the sampled insurance companies, underscoring the importance of applying efficiency analysis models to explore the sources of such variation and assess the influence of potential environmental factors.

10.2 Baseline Efficiency Assessment of Insurance Companies Using DEA

The table presents the average annual efficiency scores for the 12 insurance companies over the period 2012 to 2020, calculated using the Data Envelopment Analysis (DEA) model. The efficiency levels varied across companies and years, reflecting fluctuations in performance and productivity over time.

The results of the first-stage DEA analysis are summarized in the following table:

Table No (1)

Companies	2012	2013	2014	2015	2016	2017	2018	2019	2020	Mean
Albaraka	0.554	0.612	1	1	0.728	0.897	0.703	1	1	0.833

Alnilein	0.623	1	0.871	1	0.579	0.221	0.252	0.205	0.937	0.632
Alsalama	0.716	0.885	1	1	0.308	0.490	0.490	1	1	0.765
Islamic	0.431	1	1	0.744	0.1153	0.121	0.130	1	0.238	0.531
Juba	0.869	1	0.920	1	0.206	0.468	0.178	0.276	0.377	0.588
Savanna	1	1	1	0.921	0.267	0.574	0.574	0.602	0.448	0.709
Sheikan &Reins	1	0.218	0.501	1	0.314	1	1	1	0.740	0.760
Sudanese	0.281	1	0.540	0.379	1	0.435	0.442	1	0.639	0.635
Tawuniya	0.778	1	1	0.675	0.4116	0.295	0.305	0.304	0.510	0.586
The General	0.645	0.138	1	1	0.319	0.817	0.817	1	1	0.748
The Middle	0.367	0.300	0.862	1	0.747	1	1	0.469	1	0.749
The United	0.418	0.193	0.422	0.600	0.428	0.383	0.543	0.766	0.612	0.485

Source: Compiled by the researchers based on the output generated using **R programming language**

Some companies demonstrated relative stability in their efficiency levels. *Al Baraka Insurance* consistently achieved full efficiency in several years, notably in 2015, 2016, 2019, and 2020, indicating sustained operational efficiency. *Shikan Reinsurance* also showed remarkable improvement, reaching full efficiency in the last three years of the study period. This improvement may be attributed to the adoption of more efficient operational strategies or internal restructuring efforts that positively impacted its performance.

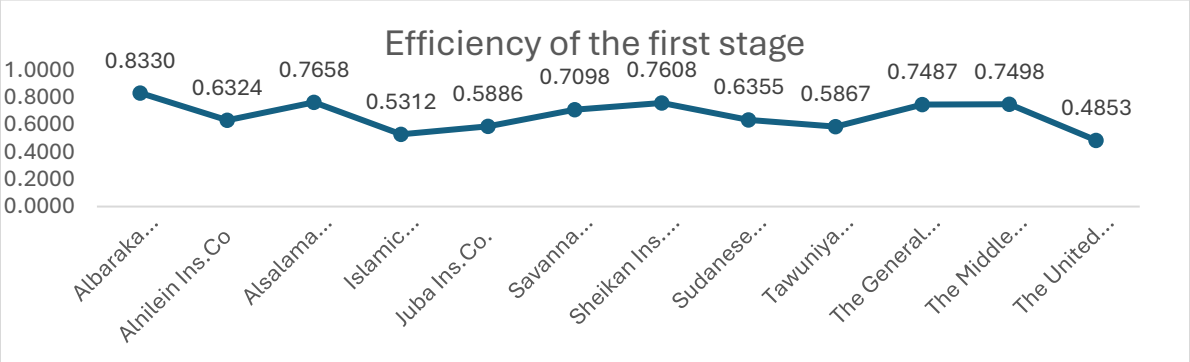
In contrast, certain companies experienced noticeable declines in efficiency. *Islamic Insurance Company*, for instance, never reached full efficiency throughout the study period and consistently recorded low scores, particularly between 2014 and 2018. This may be due to technical inefficiencies or a limited ability to allocate resources effectively. Similarly, *United Insurance Company* consistently underperformed, never exceeding an efficiency score of 0.77 in any year. Alnilein *Insurance* exhibited sharp fluctuations in performance—achieving full efficiency in years such as 2015 and 2017, but experiencing steep declines in other years. Such instability may reflect managerial inconsistency or organizational challenges, possibly linked to internal restructuring or external factors such as market shifts or regulatory changes.

It is worth noting that some companies showed gradual improvement in efficiency over time. For example, *Sudanese Reinsurance Company* displayed a steady upward trend, suggesting the long-term impact of internal development efforts. In addition, *General Insurance Company* maintained high levels of efficiency throughout most of the study period, indicating a stable operational structure and effective management practices.

These results highlight a significant disparity in operational efficiency across Sudanese insurance companies, underscoring the need for internal policy review and improved resource utilization. Furthermore, companies that consistently perform well may serve as benchmark models for sector-wide improvement.

10.3 Visualization of First-Stage Efficiency Scores for Insurance Companies

Figure (1)



Source: Compiled by the researchers based on the output generated using **R programming language**

The second figure illustrates the distribution of efficiency scores across companies in the first stage, where the values ranged from 0.4853 (the lowest efficiency recorded for *The United Insurance Company*) to 0.8330 (the highest efficiency recorded for *Albaraka Insurance Company*). This wide variation reflects significant disparities in operational performance among companies. Such differences may be attributed to variations in resources, operational scale, and the extent of external environmental influence on each company.

10.4 Statistical Summary and Distributional Tests of Adjusted Inputs and SFA Residuals

The Shapiro–Wilk test results indicate that the distributions of the adjusted inputs and most residual components deviate significantly from normality ($p < 0.05$), except for the inefficiency term (u_{ij}) and the noise component (v_{ij}), which showed closer alignment with the normal distribution. The **mean of the inefficiency component (u_{ij})** was 0.5868, suggesting a moderate level of inefficiency across the sampled insurance companies. The **noise term (v_{ij})** centered around −1, reflecting stochastic fluctuations unrelated to managerial or environmental factors. The adjusted input variables, such as capital and total rights, revealed high variance and broad ranges, indicating substantial heterogeneity even after controlling for external influences. The average first-stage efficiency score was 0.6689, serving as a baseline for comparison with post-adjustment efficiency in the third stage.

All adjusted input values, residual components, and normality test results are presented in Appendix Table A2.

10.5 SFA-Based Assessment of Environmental Effects on Managerial Inefficiency

Table (2)

maximum likelihood estimate

	Estimate	Error	z	Pr(>|z|)
Intercept	_ 2.8377e-01	0.79235	-0.35810	0.79235
Company_Age	0.8264	0.21930	0.018716	0.0041053
Company_Type_Num	0.6929	-0.39490	0.71124	-0.28089
sigmaSq	2e-16	97.571	0.40618	39.631
gamma	4.9749e	20,101	2e-16	1.0000

Source: Compiled by the researchers based on the output generated using **R programming language**

- The only variable with statistically significant impact was the age of the company (*p = 0.0041*), indicating a positive and significant relationship between company age and inefficiency. This suggests that older companies tend to exhibit higher inefficiency levels, even after controlling for environmental effects.
- The variable "type of company" was not statistically significant (*p = 0.2809*), likely due to the limited variation in this variable — with only one public company and the remainder being private — which may lead to near-perfect multicollinearity.
- The Gamma coefficient (γ ≈ 1.0000) implies that nearly all variation in inefficiency is attributable to the inefficiency term (u_{ij}) rather than the random noise component (v_{ij}). This reinforces the importance of employing a two-stage DEA model to ensure more accurate estimation of efficiency levels.
- The total variance ($\sigma^2 = 97.571$) indicates a high degree of unexplained variability in inefficiency among the companies, underscoring the necessity of environmentally

10.6 Final Efficiency Analysis After Environmental and Statistical Adjustments

Table (3)

	Stage1_Efficiency	Inefficiency Stage1	EffStage3_Efficiency	Inefficiency Stage3	Improve Range
Albaraka	0.8330	0.1670	0.9628	0.0372	0.1298
Alnilein	0.6324	0.3676	0.8894	0.1106	0.2570
Alsalama	0.7658	0.2342	0.9202	0.0798	0.1544
Islamic Insurance	0.5312	0.4688	0.8749	0.1251	0.3437
Juba	0.5886	0.4114	0.7862	0.2138	0.1976
Savanna	0.7098	0.2902	0.7384	0.2616	0.0287
Sheikan &Reins	0.7608	0.2392	0.9426	0.0574	0.1817
Sudanese & Reins	0.6355	0.3645	0.7309	0.2691	0.0954
Tawuniya	0.5867	0.4133	0.6942	0.3058	0.1075
The General	0.7487	0.2513	0.8987	0.1013	0.1500
The Middle East	0.7498	0.2502	0.7724	0.2276	0.0226
The United	0.4853	0.5147	0.8195	0.1805	0.3341

Source: Compiled by the researchers based on the output generated using **R programming language**

1. Overall Improvement in Efficiency
   All companies demonstrated improved efficiency after the implementation of the second and third stages of the DEA model. This indicates that controlling for environmental factors and statistical noise using the SFA model enhanced the accuracy of measuring true operational efficiency.

2. Greatest Improvements
   The United improved by 0.3341, and *Islamic Insurance* improved by 0.3437, suggesting that these companies were more affected by environmental variables or random noise prior to adjustment. Consequently, their first-stage efficiency scores were likely biased downward.

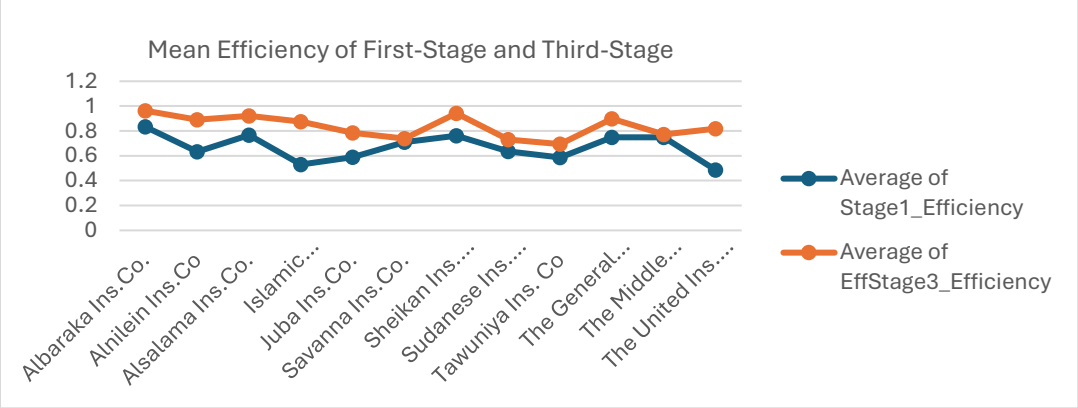
3. Smallest Improvements
   The Middle East improved by only 0.0226, and *Savanna* by 0.0287, indicating that their initial efficiency assessments were relatively accurate, with limited external influence on performance.

4. Top Performers After Adjustment (Stage 3)
   Albaraka, *Sheikan & Reins*, and *The General* achieved the highest efficiency scores following adjustment, implying these firms are operating very close to optimal efficiency.

5. Lowest Performers After Adjustment
   Tawuniya and *Sudanese Ins & Reins* showed improvements of 0.6942 and 0.7309, respectively. Despite these gains, there remains substantial room for improvement, suggesting that internal operational factors still constrain their performance.

10.7 Visualization Comparison of First-Stage and Third-Stage Efficiency Scores

Figure (2)



Source: Compiled by the researchers based on the output generated using **R programming language**

Figure 1 illustrates a comparison between the average efficiency scores estimated in Stage 1 (Efficiency Stage 1) and the adjusted averages in Stage 3 (Efficiency Stage 3). The curve clearly shows that most companies experienced improvements in efficiency levels following statistical adjustment, indicating that the initial estimates were influenced by environmental factors and statistical noise.

Notably, the Stage 3 efficiency scores significantly surpassed those of Stage 1 for most companies, highlighting the effectiveness of the SFA model in Stage 2 in enhancing the accuracy of the efficiency estimates.

The differences were especially prominent for companies such as *Islamic Insurance* and *The United Insurance*, suggesting that their performance was substantially affected by uncontrollable external factors. Conversely, the changes were relatively minor for *Savanna Insurance* and *The Middle East Insurance*, indicating stable efficiency assessments across the different stages.

11. Presentation of results

1. The results of the second stage, based on the SFA model, indicate that company age has a statistically significant and positive effect on efficiency, suggesting that firms with longer operational experience tend to achieve higher technical efficiency.
2. The type of ownership (public vs. private) did not show a significant effect on efficiency. This is likely due to the skewed distribution of this variable, as most firms in the sample were private, resulting in insufficient statistical variation.
3. The stochastic frontier model parameters revealed that the gamma coefficient (γ) was approximately equal to 1 ($\gamma \approx 1$), implying that the majority of variation in efficiency scores is attributable to inefficiency, rather than to statistical noise.
4. The adjusted input variables in Stage 2 did not follow a normal distribution based on the Shapiro-Wilk test, which justifies the use of the SFA model to correct for noise and environmental influences.
5. The estimated inefficiency components (u_{ij}) decreased after adjustment, indicating that the SFA model effectively isolated the effects of environmental and random noise from the original efficiency scores.
6. There was a clear improvement in efficiency scores after input adjustment using Stage 2 outputs, with the Stage 3 efficiency scores consistently higher than those in Stage 1.
7. The extent of improvement varied across firms, reflecting differences in their ability to utilize resources efficiently after controlling for external factors.
8. Firms with lower initial efficiency showed the greatest improvement in Stage 3, suggesting that their inefficiency was more due to external environmental factors and random disturbances than internal managerial shortcomings.

9. The study recommends the use of the Three-Stage DEA model as a primary evaluation tool in the insurance sector, particularly when comparing firm performance across heterogeneous environments.
10. The study also recommends that emerging firms be supported through targeted managerial and organizational development programs, as lack of experience was found to be a key determinant of low efficiency.

12. Recommendations

12.1 Strategic and Operational Guidelines Based on Three-Stage DEA Findings

1. Adoption of the Three-Stage DEA Model in Performance Evaluation:
   The study recommends using the Three-Stage DEA model as a standard approach for evaluating the efficiency of insurance companies, due to its ability to isolate the effects of environmental factors and statistical noise, thus providing more accurate and fair efficiency estimates.
2. Improving the Operating Environment for Less Efficient Companies:
   It is advised to develop supportive policies for companies that showed low efficiency in the first stage but significant improvement in the third stage, as this indicates their performance was strongly affected by external factors.
3. Training and Development Programs for Newly Established Companies:
   Since the SFA results showed a positive relationship between company age and efficiency, the study recommends implementing specialized administrative training and development programs to support the operational capacity of younger companies.
4. Reconsidering Resource Allocation Policies:
   Policymakers in the insurance sector should revise their resource distribution mechanisms, ensuring that environmental-adjusted efficiency levels are considered to allocate resources more effectively.
5. Adopting Flexible Strategies Based on Environmental Conditions:
   Companies should adopt adaptable operational strategies that respond to market dynamics and external conditions, especially those found to be significantly influenced by environmental variables.
6. Conducting Periodic Efficiency Assessments:
   Regular efficiency evaluations using advanced statistical models, including DEA and SFA, are recommended to monitor performance and encourage continuous improvement across the sector.
7. Enhancing the Quality of Financial and Managerial Data:
   Given the model's reliance on accurate data, it is crucial to improve internal information systems in insurance companies to ensure reliable, up-to-date data that can support sound decision-making.
8. Encouraging Transparency in Operational Performance:
   Regulatory bodies should encourage insurance companies to disclose their

efficiency evaluation results, promoting positive competition and overall performance improvement within the market.

12.2 Recommendations for Enhancing Operational Efficiency in Insurance Companies

In light of the empirical findings of this study, several actionable recommendations are proposed to support insurance companies—particularly in developing economies like Sudan—in improving their operational efficiency:

1. Modernize Management Practices in Older Firms:
   The significant association between company age and inefficiency suggests that legacy institutions should revisit their internal structures and adopt modern administrative practices. This includes investing in digital transformation, continuous staff development, and performance-based evaluation systems to reduce managerial slack.
2. Establish Internal Efficiency Monitoring Mechanisms:
   Firms are encouraged to adopt regular internal performance assessments using non-parametric methods such as DEA. This facilitates timely identification of operational weaknesses and supports data-driven decision-making processes.
3. Incorporate Environmental Factors into Strategic Planning:
   The results underscore the influence of contextual and environmental variables on firm efficiency. Insurance providers should account for factors such as market dynamics, regulatory pressures, and company structure when formulating operational strategies and allocating resources.
4. Promote Cross-Sector Collaboration and Knowledge Sharing:
   Given the dominance of private insurers and the presence of only one public company in the sample, fostering platforms for dialogue and best-practice exchange between public and private entities could enhance overall sector efficiency.
5. Implement Flexible Resource Allocation Models:
   The wide variation in resource use across companies highlights the need for more adaptable resource distribution frameworks. Companies should consider reallocating financial and human capital from underperforming segments to more productive areas, guided by efficiency metrics.

   These recommendations aim to translate the analytical outcomes of the Three-Stage DEA model into practical steps for operational improvement, contributing to a more resilient and performance-oriented insurance sector.

13. Conclusion

This study applied a robust Three-Stage DEA model, incorporating Stochastic Frontier Analysis (SFA), to evaluate the operational efficiency of Sudanese insurance companies over the period 2012–2020. The findings reveal that firm age has a significant and positive impact on efficiency, indicating that more established companies

tend to perform better. In contrast, ownership type did not show a significant effect, likely due to limited variation within the sample.

The gamma coefficient, approaching unity ($\gamma \approx 1$), confirmed that most of the variation in efficiency scores stems from inefficiency rather than random noise. Furthermore, the lack of normality in adjusted inputs justified the application of SFA in the second stage to isolate environmental and stochastic effects.

Efficiency scores improved consistently after adjustment, particularly for firms with lower initial scores, suggesting that much of their inefficiency was driven by external factors. These results highlight the importance of accounting for environmental influences when evaluating performance and underscore the value of using the Three-Stage DEA model as a fair and comprehensive benchmarking tool.

The study recommends that the model be adopted for periodic performance assessments in the insurance sector, especially in environments characterized by institutional and structural diversity. Additionally, targeted development programs should be designed to support emerging firms with limited operational experience, to enhance their technical capabilities and long-term efficiency.

14. Appendices

This section presents supplementary tables that support the main analysis and enhance the transparency and clarity of the study results. All computations were carried out using **R programming language**.

14.1 Appendix Table A1 Descriptive Analysis of Inputs, Outputs, and Environmental Variables:

	Minimum	Maximum	Mean	Std. Deviation
Capital	1000000.000000	100000000.000000	10093192.824074	10628493.572378
Total_Rights	2243252.000000	851422174.000000	27361451.298148	82517300.330105
Net_Premiums	0	670278632.000000	73428763.028056	140191712.851414
Total_Direct_Prememiums_For_Takaful	0	61024888.000000	3809749.703704	11643789.046210
Investment_Deposits_in_Bank	0	60993834.000000	5331232.287037	11400223.205478
Outward_Premiums	2243252.000000	404634588.000000	38795324.629630	66907519.165018
Loss_Ratioper	293000.000000	100750000.000000	6020885.568648	13644921.786895
Profit	293000.000000	2110000000.000000	35181155.203704	202192476.337138
Company_Age	32	63	44.250000	9.263661
Company_Type_Num	0	1	0.08	0.277674

This table A1 descriptive statistics table summarizes the inputs, outputs, and environmental factors used in the three-stage DEA model. The data reveal substantial variation in values across the studied companies, indicating differing operational scales and environmental conditions.

14.2 Appendix Table A2 Descriptive Statistics and Normality Tests for Adjusted Inputs and Residual Components (Second Stage)

	mean	min	max	std	Sigma^2 (Variance)	shapiro_p
Capital_Adjusted	10093193	999999.63	100000000.4	10628494	1.12965E+14	4.72015E-17
Total_Rights_Adjusted	27361451	2243251.709	851422173.7	82517300	6.8091	1.54968E-21
Net_Premiums_Adjusted	73428763	-0.287761861	670278632.3	1.4E+08	1.9653	1.71745E-16
Total_Direct_Prememiums_For_Takaful_Adjusted	3809750	-0.370009279	61024888.34	11643789	1.35578E+14	1.7881E-19
SFA_residual	3.9E-16	-0.373346661	0.586841712	0.298563	0.089139663	6.37665E-07
Stage1_Efficiency	0.6689	0.1153	1	0.301025	0.09061	3.288
u_ij	0.5868	0	0.9601	0.298563	0.0891	6.376
v_ij	-1	-1.3733	-0.4131	0.298559 7	0.2985	5.376

This table A2 presents the statistical characteristics of the adjusted variables subjected to analysis in the second stage of the Three-Stage DEA model, employing the Stochastic Frontier Analysis (SFA) approach. It includes the Shapiro-Wilk test values to assess the normality of the distributions, in addition to variance (Sigma²), standard deviation, maximum and minimum values, and the arithmetic mean for each variable.

The results indicate that the adjusted input variables (such as **Capital_Adjusted** and **Total_Rights_Adjusted**) experienced notable changes in their mean and dispersion following the application of the SFA model, reflecting the model's effectiveness in isolating the effects of environmental and random noise factors. The table also includes the residuals from the SFA model (**SFA_residual**), the inefficiency term (**u_ij**), the statistical noise term (**v_ij**), and the efficiency scores from the first stage (**Stage1_Efficiency**), in order to illustrate the quantitative shifts that occurred after adjustment.

The presence of extreme values in some variables (e.g., the maximum value for **Net_Premiums_Adjusted**) reveals substantial heterogeneity among the observed insurance companies, thereby justifying the need for a multi-stage model to control for exogenous factors beyond managerial influence. Furthermore, the Shapiro-Wilk test results suggest that most variables do not follow a normal distribution at the conventional significance level ($\alpha = 0.05$), supporting the appropriateness of applying non-parametric and multi-stage estimation techniques.

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